Math 441 - Homework 3 Due Wednesday, Sept. 28

1. Suppose that $x_1, x_2, ..., x_n$ are real numbers. Prove that

$$|x_1 + x_2 + ... + x_n| \leq |x_1| + |x_2| + ... + |x_n|.$$

(Hint: Use induction. You may also want to use the triangle inequality.)

2. Let $y > 0$. Prove that there exists a *unique* $n \in \mathbb{N}$ such that $n - 1 \leq y < n$. (Hint: You need to prove existence and uniqueness separately. To prove uniqueness use proof by contradiction.)

3. Let $S \subseteq \mathbb{R}$ and $x \in \mathbb{R}$. Prove that one and only one of the following conditions holds:

   (a) $x \in \operatorname{int} S$

   (b) $x \in \operatorname{int}(\mathbb{R} \backslash S)$

   (c) $x \in \operatorname{bd} S = \operatorname{bd}(\mathbb{R} \backslash S)$