Math 441 - Homework 4

1. (20 points) Let S and T be subsets of $\mathbb{R}$. Prove the following.

   (a) $\operatorname{cl}(\operatorname{cl} S) = \operatorname{cl} S$.

   (b) $\operatorname{cl}(S \cup T) = (\operatorname{cl} S) \cup (\operatorname{cl} T)$.

   (c) $\operatorname{cl}(S \cap T) \subseteq (\operatorname{cl} S) \cap (\operatorname{cl} T)$.

   (d) Find an example to show that equality need not hold in part (c).

2. (15 points) Let S be a subset of $\mathbb{R}$. Prove that S is compact if and only if every infinite subset of S has an accumulation point in S.

3. (15 points) Let (X, d) be a metric space. We say that a subset $D \subseteq X$ is **dense** in X if $\operatorname{cl} D = X$. Prove that D is dense in X iff every nonempty open subset of X intersects D.