Math 441 - Homework 7

1. (16 points) Let (a_n) and (b_n) be bounded sequences.

   (a) Prove that $\liminf a_n + \liminf b_n \leq \liminf(a_n + b_n)$.

   (b) Find an example to show that equality may not hold in part (a).

2. (10 points) Use the $\epsilon - \delta$ definition to prove that $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x - 1} = 2$

3. (10 points) Let $f : \mathbb{R} \to \mathbb{R}$ and suppose that $\lim_{x \to c} f(x) > 0$. Prove that there exists a deleted neighborhood U of c such that $f(x) > 0$ for all $x \in U$.