**Math 111 - Midterm 3 Review Problems**

*Most of these problems do not need a calculator. The ones where a calculator is needed say so. Remember, you will not be allowed to use a calculator on the test.*

1. In each of the following weighted voting systems, determine which players, if any, are dictators or dummies (remember a dictator has all the power and a dummy has none).
   (a) $[13 : 15, 3, 2]$
   (b) $[13 : 8, 5, 4, 3]$
   (c) $[17, 8, 5, 4, 3]$

2. A person in a weighted voting system has veto power if a decision cannot be made without their approval. Can more than one person have veto power in a weighted voting system? If so, give an example, if not, explain why not.

3. In each of the following weighted voting systems, find the Banzhaf power distributions of the players.
   (a) $[13 : 15, 3, 2]$
   (b) $[13 : 8, 5, 4, 3]$
   (c) $[17, 8, 5, 4, 3]$

4. Explain why a dummy cannot be a member of every winning coalition.

5. The honor council at Big State University is composed of five members, two faculty members and three students. To make a decision requires three votes, at least one of which must be from a faculty member.
   (a) List all winning coalitions.
   (b) In each winning coalition, circle all of the critical players.
   (c) What is the Banzhaf power distribution of the honor council?

6. Suppose there are 5 main courses on a menu and the main courses each come with a choice of three <u>different</u> side dishes chosen from the following options: bake potato, baked beans, green beans, corn, applesauce, house salad. How many dinners are possible?

7. If you flip a fair coin 400 times, the average number of heads is about 200 with a standard deviation of 10 heads. What is the average numbers of heads you will get if you flip the coin 40,000 times? What is the standard deviation?

8. Why do your odds of breaking even get lower as you play more and more games against the house at a casino?

9. A box contains three red balls, two blue balls, and one green ball. A ball is drawn at random. A second ball is then drawn at random. The balls are not put back in the box.
   (a) Show all the possible outcomes using a weighted tree diagram.

(b) What is the probability that you draw a red ball both times?

(c) Calculate the probability of getting one blue and one red ball.

10. Suppose that you flip an unfair coin that lands on heads 60% of the time. Make a weighted tree diagram for the possible outcomes of flipping the coin twice, and find the probability of each outcome. Make sure that you show the weights on each edge.

11. **Calculator Problem.** What is the middle entry in the 18th row of Pascals triangle? (Hint the middle entry is entry 9, if you start counting from zero).

12. Find the following without a calculator.

(a) ${}_6C_4$

(b) ${}_6P_3$

(c) ${}_{40}C_{38}$

13. The United States Senate currently has 55 Republicans, 44 Democrats, and 1 Independent. You may write your answer using the ${}_nC_k$ notation.

(a) How many different committees of 6 senators are possible?

(b) How many different 6-person committees with no Republicans are possible?

(c) If a committee of 6 senators were selected at random, how likely would it be for the committee to contain no Republicans?

14. **Calculator Problem.** The scoring for a college course is given in the following table.

| | Exam 1 | Exam 2 | Exam 3 | In-Class | Paper | Final Exam |
|---|---|---|---|---|---|---|
| Weight | 15% | 15% | 15% | 10% | 25% | 20% |
| Bob's scores | 77 | 83 | 91 | 90 | 87 | ? |

What grade would Bob need on the final exam to get an 83 in the course?

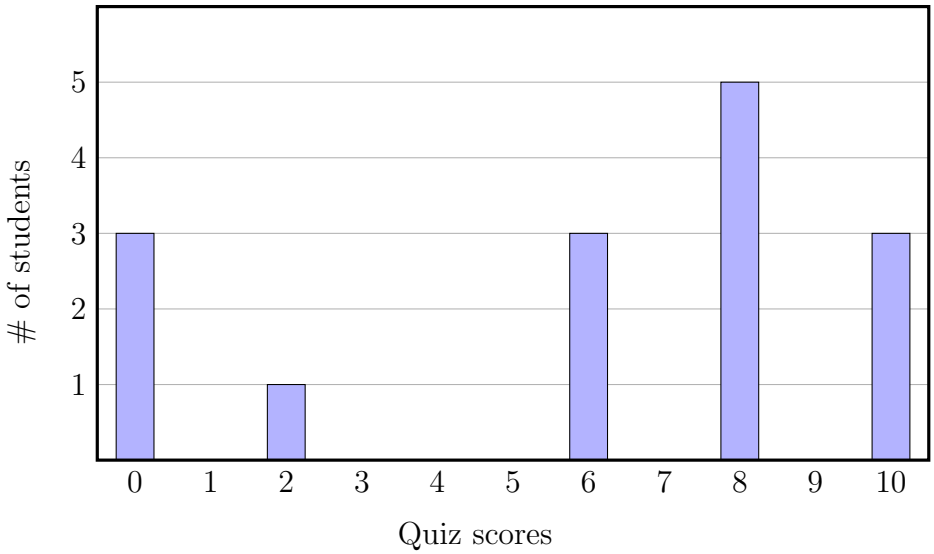
15. Find the expected value of a random variable with four possible outcomes and probabilities shown below.

| Outcome | -1 | 0 | 4 | 10 |
|---|---|---|---|---|
| Probability | 0.1 | 0.25 | 0.25 | ? |

16. A class of history students received the following quiz scores. Draw a histogram for the data below.

| Score | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Frequency | 0 | 0 | 2 | 0 | 0 | 3 | 2 | 3 | 5 | 2 | 3 |

17. Use the histogram below to find the average of the students' quiz scores.




18. A cannery produces cans of fruit that are supposed to weigh 12 ounces, but not every can weighs exactly 12 ounces. The average is actually 12.5 ounces and the standard deviation is 0.5 ounces and the weight of the cans has an approximately normal distribution.

    (a) Draw a bell curve that represents the distribution of the weights of cans. Be sure to label the $x$-axis.

    (b) What percent of cans will weigh 12 ounces or more?

    (c) What percent of cans will weigh less than 11 ounces?