Math 441 - Homework 2   **Due Friday, Sept. 13th**

1. (10 points) Is the relation $x \equiv_k y$ iff $x + y$ is divisible by k an equivalence relation on $\mathbb{Z}$? Explain why or why not. Which of the properties (Transitivity, Symmetry, Reflexivity) does it satisfy?

2. (10 points) A relation R on a set S is **circular** iff xRy and yRz implies that zRx, for all $x, y, z \in S$. Prove that a relation is circular and reflexive if and only if it is an equivalence relation.

3. (5 points) Draw the following subsets of the Cartesian plane $\mathbb{R}^2$.

   (a) $[3, 5] \times [2, 4)$

   (b) $\{1, 2, 3\} \times (0, 1)$

   If you draw the images on the same axis, make sure you label each region.

4. (5 points) Give examples of relations with the following properties.

   (a) Reflexive, but not symmetric and not transitive.

   (b) Symmetric, but not transitive and not reflexive.

   (c) Reflexive and symmetric, but not transitive.