Math 441 - Homework 3 **Due Friday, Sept. 20th**

1. (10 points) Suppose the $f : A \to B$ and $S, T \subseteq A$. Prove or give a counterexample.
   (a) $S \subseteq T \Rightarrow f(S) \subseteq f(T)$.

   (b) $f(S) \subseteq f(T) \Rightarrow S \subseteq T$.

2. (10 points) Use induction to prove: If $1 + x > 0$, then $(1 + x)^n \geq 1 + nx$ for all $n \in \mathbb{N}$.

3. (5 points) Prove that $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n + 1)(2n + 1)$ for all $n \in \mathbb{N}$.

4. (5 points) Show that the following pairs of sets A, B are equinumerous by describing a specific bijection between the sets in each pair.
   (a) $A = \mathbb{N}$ and B is the set of all odd integers.

   (b) $A = (0, 1)$ and $B = (1, \infty)$.

   (c) (*For 3 points of extra-credit*) $A = (0, 1)$ and $B = [0, 1]$. (See the hint for problem 8.3(b) in the back of the book.)