Math 242 **Final Exam Review**

1. Given the three vectors $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - \mathbf{k},\ \mathbf{b} = \mathbf{i} + \mathbf{k},$ and $\mathbf{c} = \mathbf{j} + 2\mathbf{k}$
   (a) Find the length of $\mathbf{a}$ and find a unit vector in the direction of $\mathbf{a}$.
   (b) Find a vector that is orthogonal to both $\mathbf{b}$ and $\mathbf{c}$
   (c) Determine whether the three vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ are colinear or not.
2. Find the equation of the plane that passes through the point $(1, 2, 3)$ and contains the line $x = 3t, y = 1 + t, z = 2t$.
3. Find the unit tangent vector $\mathbf{T}(t)$ to the parametric curve $\mathbf{r}(t) = \cos t\mathbf{i} + 3t\mathbf{j} + 2\sin 2t\mathbf{k}$ at the point where $t = 0$.
4. Find the tangent plane to the elliptic paraboloid $z = x^2 + 3y^2$ at the point $P = (1, 1, 4)$.
5. Use implicit differentiation to find $\partial z/\partial x$ and $\partial z/\partial y$, where $x^2 + y^2 + z^2 = 3xyz$.
6. Consider the function $f(x, y) = 2\sqrt{x} - y^2$.
   (a) Find the directional derivative of f at the point $(1, 2)$ in the direction $v = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.
   (b) Find the maximum rate of change of f at the point $(1, 2)$. In what direction does it occur?
7. Evaluate the double integral below, over the region $D = \{(x, y) \mid 0 \le x \le 1, 0 \le y \le \sqrt{x}\}$,
$$\iint \frac{2y}{1 + x^2}\, dA.$$
8. Evaluate the triple integral below, where E lies above the $z = 0$ plane, below the plane $z = y$ and inside the cylinder $x^2 + y^2 = 4$:
$$\iiint_E yz\, dV$$
9. Evaluate the line integral $\displaystyle\int_C F \cdot d\mathbf{r}$, where $F(x, y, z) = x\mathbf{i} - z\mathbf{j} + y\mathbf{k}$ and the curve C is parametrized by $\mathbf{r}(t) = 2t\mathbf{i} + 3t\mathbf{j} - t^2\mathbf{k}$, $0 \le t \le 1$.
10. Show that the line integral is independent of path and evaluate it:
$$\int_C (1 - ye^{-x})\, dx + e^{-x}\, dy,\ C \text{ is any path from } (0, 1) \text{ to } (1, 2).$$
11. Use Green's Theorem to evaluate the line integral
$$\int_C y^3 dx - x^3 dy,$$
C is the circle $x^2 + y^2 = 4$ with counterclockwise orientation.
12. Evaluate $\displaystyle\int_C ye^x\, ds$ where C is the line segment joining $(1, 2)$ to $(4, 7)$.