Math 242 - Homework 13 — Due Thursday, December 4

Exercises from the Book

- Chapter 4.1# 1, 2, 4, 5, 6, 7, 13
- Chapter 4.2# 1, 2, 3, 4
- See Page 191 for solutions to selected problems.

Additional Exercises

1. Use the formula $W = \int_C F \cdot d\mathbf{r}$ to find the amount of work W done by the force field $F(x, y) = x^2\mathbf{i} - xy\mathbf{j}$ as a particle moves along the curve C given by the vector-valued function $\mathbf{r}(t) = (\cos t, \sin t)$ from $t = 0$ to $t = \pi/2$. (Answer: $-2/3$)

2. Suppose that an object with mass M is located at the origin and another object with mass m is located at $\mathbf{r} = (x, y, z)$ in $\mathbb{R}^3$. The force of gravity acting on the second object is
$$F_G = \frac{-mMG}{||\mathbf{r}||^3}\mathbf{r},$$
where G is the gravitational constant. Show that F_G is a conservative vector field by showing that the function
$$f_G = \frac{mMG}{\sqrt{x^2 + y^2 + z^2}}$$
is a potential function for F_G (i.e., show that $F_G = \nabla f_G$).

3. Is $F(x, y, z) = y^2\mathbf{i} + (2xy + e^{3z})\mathbf{j} + 3ye^{3z}\mathbf{k}$ a conservative vector field? If so, what is the potential function f?