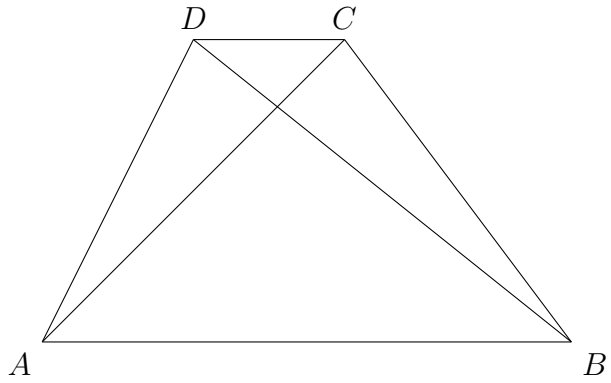
Math 242 **Midterm 1 Review**

1. (8 points) Write each combination of vectors in the figure below as a single vector.



   (a) $\overrightarrow{DC} + \overrightarrow{CB}$

   (b) $\overrightarrow{AD} - \overrightarrow{AC}$

2. Find the angle between the vectors $\mathbf{a} = (1, 2, 3)$ and $\mathbf{b} = (-2, 0, 1)$. It is OK to give a formula for the angle.

3. Find the area of the triangle $P = (2, 1, 7), Q = (1, 1, 5)$, and $R = (2, -1, 1)$. Hint: The area of a triangle is half of the area of a parallelogram.

4. Convert the following points in polar coordinates to rectangular coordinates.

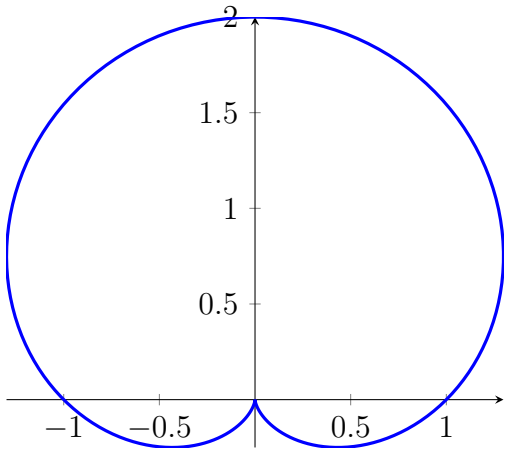
   (a) $(r, \theta) = (-2, \pi/2)$,

   (b) $(r, \theta) = (\sqrt{2}, 3\pi/4)$.

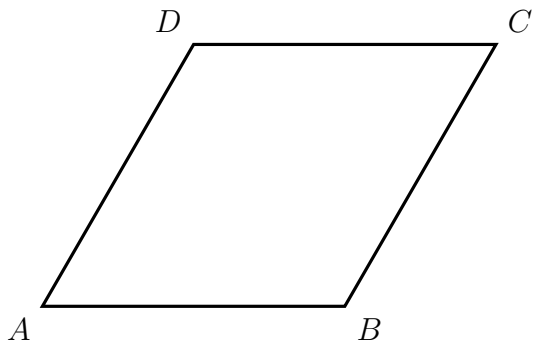
5. (a) Sketch the curve defined by the parametric equations $x(t) = \sin t$ and $y(t) = t^2$ where $0 \leq t \leq 2\pi$. Indicate with an arrow what direction the curve is traced as t increases.

   (b) Find a formula for the slope of the tangent lines as a function of t.

6. Find the area inside the polar curve $r = 1 + \sin\theta$ (graph shown below).



7. Show that the diagonals of a rhombus are always perpendicular. Hint: let $\mathbf{x} = \overrightarrow{AB}$ and $\mathbf{y} = \overrightarrow{AD}$.



8. Let $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{b} = \mathbf{j} + 2\mathbf{k}$. Compute the values of each expression, or explain why the expression does not make sense.
   (a) $||\mathbf{a}||$
   (b) $\mathbf{a} \cdot \mathbf{b}$
   (c) $\mathbf{b}^2$
   (d) $\mathbf{a} \times \mathbf{b}$
   (e) $||\mathbf{a}|| \times \mathbf{b}$

Formulas

Arc Length for Parametric Curves

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Areas in Polar Coordinates

$$A = \int_a^b \tfrac{1}{2} r^2 d\theta$$

Cross Product

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$