Math 242 **Midterm 2 Review**

1. Express the curve $y = x^2$ using a vector valued function (Hint: use x as the parameter).

2. Write a formula for the tangent line at $(1, 1, 1)$ for each of the curves $\mathbf{r}_1 = (t, t^2, t^3)$ and $\mathbf{r}_2(u) = (1, \sqrt{2}\sin u, \sqrt{2}\cos u)$.

3. Show that $\dfrac{d}{dt}\mathbf{r}(t) \times \mathbf{r}'(t) = \mathbf{r}(t) \times \mathbf{r}''(t)$.

4. Compute the arc length of the curve $\mathbf{r}(t) = (2t, \frac{4}{3}t^{3/2}, \frac{1}{2}t^2)$ on the interval $1 \le t \le 3$.

5. Find the velocity, acceleration, and speed of a particle with the given position function. Sketch the path of the particle and draw the velocity and acceleration vectors for the specified value of t.
   (a) $(t^2 - 1, t)$ at $t = 1$,
   (b) $e^t\mathbf{i} + e^{-t}\mathbf{j}$ at $t = 0$.

6. For $\mathbf{r}(t) = (2\sin t, 5t, 2\cos t)$, find the unit tangent and unit normal vectors $\mathbf{T}(t)$ and $\mathbf{N}(t)$, then calculate the curvature $\kappa(t)$.

7. Sketch the curve given by the vector valued function $(1, -2) + \cos(t)(1, 0)$.

8. Find the center and radius of the sphere $x^2 + 6x + y^2 + 4y + z^2 + 2z = -10$.

9. Is the curve $(t^2, 3t, 9t - 2t^2)$ completely contained in the plane with normal vector $\mathbf{n} = (2, -6, 1)$ passing through the origin? Explain why or why not.

10. A spaceship is travelling with acceleration $\mathbf{a}(t) = (e^t, t, \sin(2t))$. At time $t = 0$ the spaceship is at the origin $\mathbf{r}(0) = (0, 0, 0)$ with initial velocity $\mathbf{v}(0) = (1, 0, 0)$. Find the position of the spaceship $t = \pi$.

11. Complete the following coordinate conversion table:

	to rectangular	to cylindrical	to spherical
from rectangular	$x = x$ $y = y$ $z = z$		
from cylindrical	$x = r\cos\theta$ $y = r\sin\theta$ $z = z$		
from spherical			$\rho = \rho$ $\theta = \theta$ $\phi = \phi$

12. Find a rectangular equation for the surface whose spherical equation is $\rho = \sin\theta\sin\phi$.