Math 242

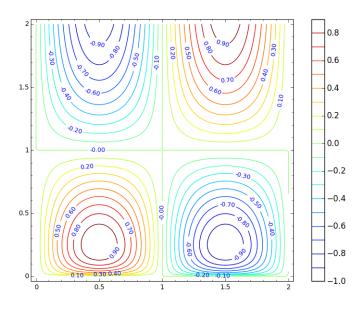
Midterm 4 Review

1. For each of the following integrals, draw a picture of the region in \mathbb{R}^2 being integrated over.

(a)
$$\int_{-2}^{0} \int_{0}^{\sqrt{4-x^2}} f(x,y) \, dy \, dx.$$

(b) $\int_{\pi/4}^{3\pi/4} \int_{-1}^{1} g(r,\theta) \, r \, dr \, d\theta$
(c) $\int_{0}^{1} \int_{2x}^{2} h(x,y) \, dy \, dx.$

- 2. Reverse the order of the integrals to compute $\int_0^1 \int_{2x}^2 e^{y^2} dy dx$. Hint: you will need to change the bounds if you integrate with respect to x first.
- 3. Find the volume of the solid whose base is the region in the xy-plane that is bounded by the parabola $y = 4 x^2$ and the line y = 3x, while the top of the solid is bounded by the plane z = x + 4.
- 4. Use a Riemann sum of the form $\sum \sum f(x_i^*, y_j^*) \Delta x \Delta y$ to estimate $\iint_R f(x, y) dxdy$ where $f(x, y) = \sin(\pi x) \sin(\pi \sqrt{y})$ is the function shown in the contour plot below and $R = [0, 2] \times [0, 2]$. Use 4 sub-rectangles for your estimate.



- 5. Evaluate $\iint_D y \, dA$ where D is the region between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ in the first quadrant.
- 6. Find the volume of the region between the planes x = 0, y = 0, z = 0, and 4x + 3y + 2z = 12.