Homework 3 **Quantum Computing**

Fact. Normal matrices $A, B \in M_n$ commute if and only if they are both diagonalizable by the same unitary matrix U.

For two operators $A, B \in M_n$, the **commutator** of A and B is:

$$[A, B] = AB - BA$$

Exercises

1. Show that $[A, B] = 0$ if and only if A and B commute.

2. Let
$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$
be the Pauli matrices. Compute the following: $[\sigma_1, \sigma_2]$, $[\sigma_2, \sigma_3]$, and $[\sigma_1, \sigma_3]$.

3. Compute $e^{i\theta\sigma_1}$. Hint, diagonalize σ_1 first. Then compare with (2.31) in section 2.2.1 of the lecture notes.

4. Prove that $\lim_{n\to\infty} \left(1 + \frac{z}{n}\right)^n = e^z$ for any $z \in \mathbb{C}$.