## Math 441 - Final Exam (Take-Home Portion) Due Tuesday, Dec. 14

This part of the exam is due on Tuesday, Dec. 14 at 11:59pm. It will be submitted and graded just like the homework. These problems are open book and you are free to use your notes and old homework (including my comments). Do not consult the internet or other students. Ask me if you have any questions or feel like you need a hint. I will keep the scores from the best three out of four of these problems.

- 1. Prove that not all infinite intersections of open sets are open.
- 2. If  $f : [a, b] \to \mathbb{R}$  is continuous and for some  $c \in (a, b)$ , f(c) > 0, prove that there is a neighborhood U around c such that f(x) > 0 for all  $x \in U$ .
- 3. A function is **increasing** on an interval I if for all  $x_1 < x_2$  in I,  $f(x_1) \ge f(x_2)$ . Prove that if f is differentiable and  $f'(x) \ge 0$  for all  $x \in I$ , then f(x) is increasing on I.
- 4. Suppose that f is integrable on [a, b] and that  $[c, d] \subseteq [a, b]$ . Prove that f is integrable on [c, d].

The in-class portion of the final exam is on Tuesday, Dec. 14 at 2pm. You will not be able to use your book or notes during the in-class portion. I have posted an outline of the topics that will be covered on my website:

http://people.hsc.edu/faculty-staff/blins/fall15/math441/