Math 441 - Homework 10 **Due Friday, Nov. 13**

1. Let $f : D \to \mathbb{R}$ and define $|f| : D \to \mathbb{R}$ by $|f|(x) = |f(x)|$. Suppose that f is continuous at $c \in D$. Prove that $|f|$ is continuous at c.

2. Let K be a nonempty compact subset of $\mathbb{R}$ and let $p \in \mathbb{R}$. Prove that K has a "closest" point to p. That is, prove that there exists a point $q \in K$ such that $|q-p| = \inf\{|x-p| : x \in K\}$. Hint: *Using the previous problem, observe that the function that maps x to $|x-p|$ is continuous. What do we know about continuous functions and compact sets?*

3. Suppose that $f : [a, b] \to [a, b]$ is continuous. Prove that f has a **fixed point**. That is, prove that there exists $c \in [a, b]$ such that $f(c) = c$. Hint: *Consider the function $g(x) = f(x) - x$.*