Math 441 - Homework 4 **Due Friday, Sept. 25th**

1. Suppose that $x_1, x_2, ..., x_n$ are real numbers. Prove that

$$|x_1 + x_2 + ... + x_n| \leq |x_1| + |x_2| + ... + |x_n|.$$

   (Hint: Use induction. You may also want to use the triangle inequality.)

2. Let y be any positive real number. Prove that there exists a *unique* $n \in \mathbb{N}$ such that $n - 1 \leq y < n$.
   (Hint: You need to prove existence and uniqueness separately. To prove uniqueness use proof by contradiction.)

3. Suppose that $f : A \to B$ is onto. Use the Axiom of Choice (see below) to prove that $|B| \leq |A|$. (Hint: prove that there is a one-to-one map from B to A.)

   Axiom of Choice Given any family of nonempty sets $\{S_i\}_{i \in I}$, indexed by a set I, there is a function $f : I \to \bigcup_{i \in I} S_i$ such that $f(i) \in S_i$ for every $i \in I$.