Math 441 - Homework 9

1. Use the ϵ - δ definition of limit to prove that

$$\lim_{x \to 3} \frac{2}{x+3} = \frac{1}{3}.$$

- 2. Prove that all polynomials are continuous everywhere on \mathbb{R} . Hint: Use Theorem 21.10 to prove that all 1st degree polynomials are continuous, and then use induction.
- 3. Let $f : \mathbb{R} \to \mathbb{R}$ be any function. Let $\{G_{\alpha}\}_{\alpha}$ be any collection of subsets of \mathbb{R} . Prove that $f^{-1}(\bigcup_{\alpha} G_{\alpha}) = \bigcup_{\alpha} f^{-1}(G_{\alpha})$. Hint: Use the definitions of pre-images and unions. We will use this result to prove that continuous functions map compact sets to compact sets.