Calculus I - Math 141 **Midterm 1**

Name: _____

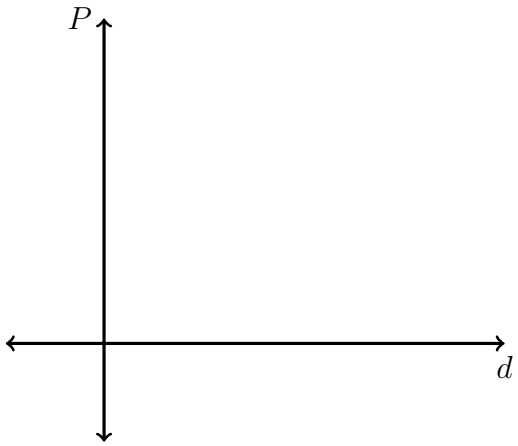
You must show all work to earn full credit. No calculators allowed. If you do not have room in the given space to answer a question, use the back of another page and indicate clearly which work goes with which problem.

Problem	Maximum Points	Your Score
1	12	
2	10	
3	8	
4	10	
5	20	
6	6	
7	10	
8	8	
9	16	
Total:	100	

1. (12 points) As you go deeper and deeper in the ocean, the pressure increases linearly. At the surface, pressure is 1 atmosphere (ATM). For every 10 meters underwater, pressure increases by 1 additional ATM.

   (a) Find a formula for pressure (P) as a function of depth underwater (d).

   (b) Draw and label a graph of pressure vs. depth.



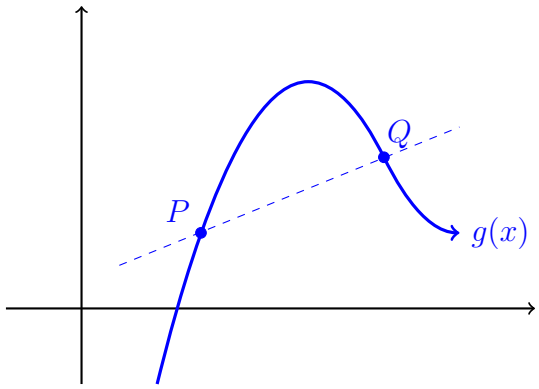
   (c) What is the derivative of the pressure function? Your answer should be a number. Explain what this number represents and what units are used to measure it.

2. (10 points) A rock thrown up in the air has height $h(t) = 8t - 16t^2 + 4$ measured in feet at time t seconds.

   (a) Find a formula for the velocity of the rock as a function of time.

   (b) Find a formula for the acceleration of the rock.

3. (8 points) The graph of a function $g(x)$ is shown below. The slope of the secant line that passes through points P and Q is represented by the difference quotient $\dfrac{g(b)-g(a)}{b-a}$.



(a) Which part of the difference quotient represents the x-coordinate of P?

(b) Which part of the difference quotient represents the y-coordinate of P?

(c) Which part of the difference quotient represents the x-coordinate of Q?

(d) Which part of the difference quotient represents the y-coordinate of Q?

4. (10 points) Use the limit definition of the derivative $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ to find the derivative of $f(x) = \dfrac{1}{2x}$.

5. (20 points) Find the following derivatives using any valid method. You don't have to simplify your answers.

   (a) $\dfrac{d}{dx}\, 2x^6 - 5x^3 + 5x$

   (b) $\dfrac{d}{dx}\, \dfrac{\tan x}{\sin x}$

   (c) $\dfrac{d}{dt}\sqrt{t}(5 - \sqrt{t})$

   (d) $\dfrac{d}{dr}\, \dfrac{4}{r^3}$

   (e) $\dfrac{d}{dx}\, \sec(4x)$

6. (6 points) Suppose that f and g are functions such that $f(5) = 3$, $g(3) = -4$, $f'(5) = 2$, and $g'(3) = \frac{1}{2}$.

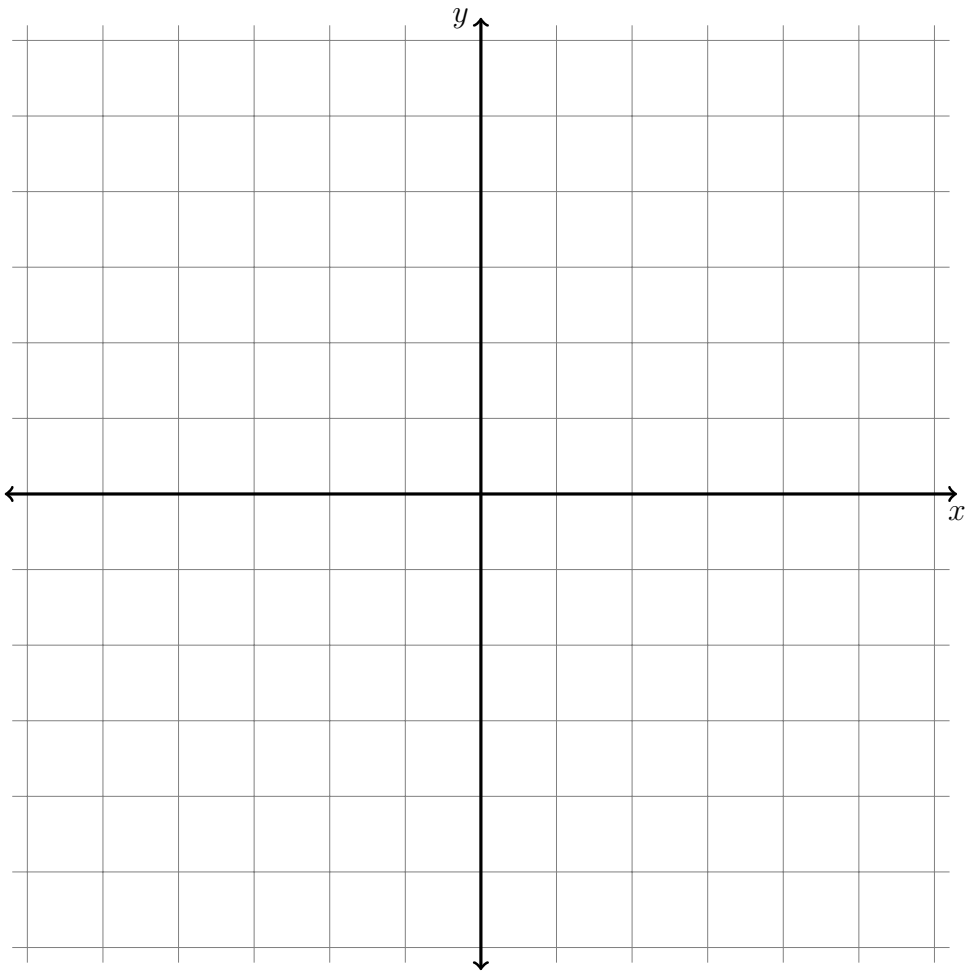
   (a) Find $g(f(5))$.

   (b) Find the derivative of $g(f(x))$ at $x = 5$.

7. (10 points) Use the axes below to sketch a graph of the parabola

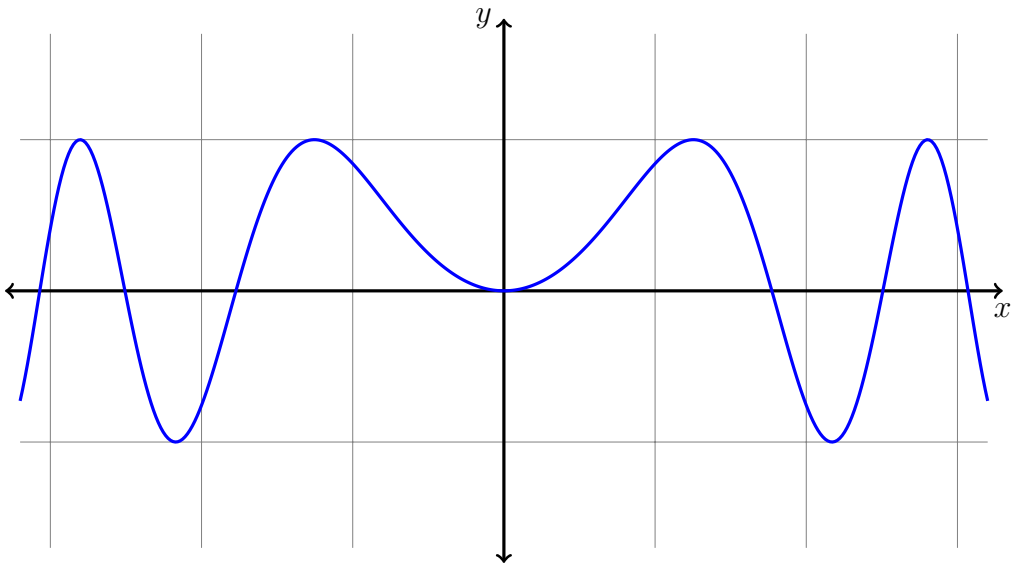
$$y = -x^2 + 4x - 3.$$

Include the coordinates of the vertex and the roots of the parabola in your graph.



8. (8 points) Find a formula for the tangent line to the parabola above at the point $(0, -3)$. Once you find this tangent line, add its graph to the graph above.

9. (16 points) The function $f(x) = \sin(x^2)$ has the graph shown below.



(a) Is this function even, odd, or neither?

(b) Use the chain rule to find $f'(x)$.

(c) Find the x-coordinates of all the points where $f(x)$ has a horizontal tangent.

Formula Sheet

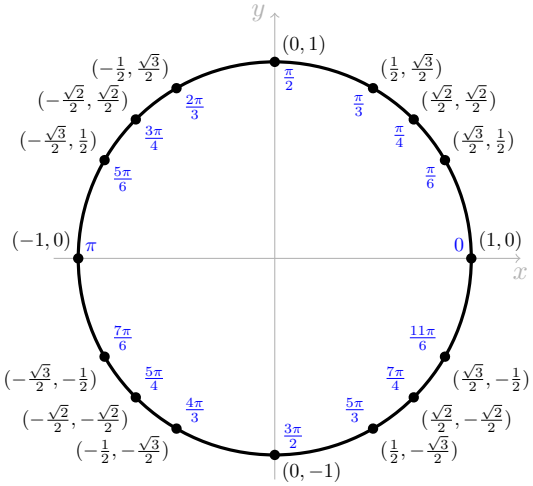
Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Point-Slope Form

$$y - y_1 = m(x - x_1)$$

Common Angles



Trigonometry Ratios

- $\tan x = \dfrac{\sin x}{\cos x} \qquad \cot x = \dfrac{\cos x}{\sin x}$
- $\sec x = \dfrac{1}{\cos x} \qquad \csc x = \dfrac{1}{\sin x}$

Trig Identities

- $\cos(a + b) = \cos a \cos b - \sin a \sin b$
- $\sin(a + b) = \sin a \cos b + \sin b \cos a$

Selected Derivatives

- $\dfrac{d}{dx} \tan x = \sec^2 x$
- $\dfrac{d}{dx} \sec x = \sec x \tan x$
- $\dfrac{d}{dx} \cot x = -\csc^2 x$
- $\dfrac{d}{dx} \csc x = -\csc x \cot x$

Definition of Derivative

- $\lim\limits_{h \to 0} \dfrac{f(x + h) - f(x)}{h}$, or
- $\lim\limits_{x \to c} \dfrac{f(x) - f(c)}{x - c}$

Linear Approximation

- $L(x) = f(a) + f'(a)(x - a)$

Summation Formulas

- $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
- $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

Riemann Sum

- $A = \lim\limits_{n \to \infty} \displaystyle\sum_{i=1}^{n} f(x_i) \Delta x$