Homework 5 - Computer Science 461 **Name:** ______________________

Due Monday, October 2.

1. Construct a context free grammar that generates each of the following languages.
   (a) $\{a^{2n}b^n : n \in \mathbb{N}\}$

   (b) $\{w \in \{0,1\}^* : w$ starts and ends with the same symbol.$\}$

   (c) $\{w \in \{a,b,c\}^* :$ length of w is odd and its middle symbol is $b\}$

   (d) $\{w \in \{0,1\}^* : w$ is a palindrome.$\}$ Hint: *Make sure your grammar generates both even and odd length palindromes.*

2. Identify the parts of the tuple (V, Σ, R, S) in your answer to problem 1 part (b).

3. Let $\Sigma = \{\texttt{(},\texttt{)},\texttt{[},\texttt{]}\}$. That is, Σ is the alphabet consisting of the four symbols `(`, `)`, `[`, and `]`. Let L be the language over Σ consisting of strings in which both parentheses and brackets are balanced. For example, the string `([][()()])([])` is in L but `[(])` is not. Find a context-free grammar that generates the language L.

4. Show that the following grammar is ambiguous by finding a string that has two different left derivations.

$$
\begin{aligned}
S &\to SS \\
S &\to aSb \\
S &\to bSa \\
S &\to \epsilon
\end{aligned}
$$

5. Draw two different parse trees for the string *ababbaab* based on the grammar in the previous problem.

6. Suppose that the string *abbcabac* has the following parse tree, according to some grammar G. Identify 5 production rules that must be rules in the grammar G.