Calculus II - Math 142 **Final Exam Review Problems**

1. Evaluate the following integrals.

   (a) $\int e^x \cos(e^x)\, dx$.

   (b) $\int \tan^5\theta \sec^3\theta \, d\theta$.

   (c) $\int x^2 \cos(3x)\, dx$

2. Find the third degree Taylor polynomial for $f(x) = x^3 + 2x - 3$ centered at $c = 2$.

3. Solve the differential equation $\frac{dy}{dx} = \frac{\cos x}{y^2}$ with initial condition $y(\pi) = 2$.

4. For each of the following series, determine whether it converges or diverges and give your reasoning.

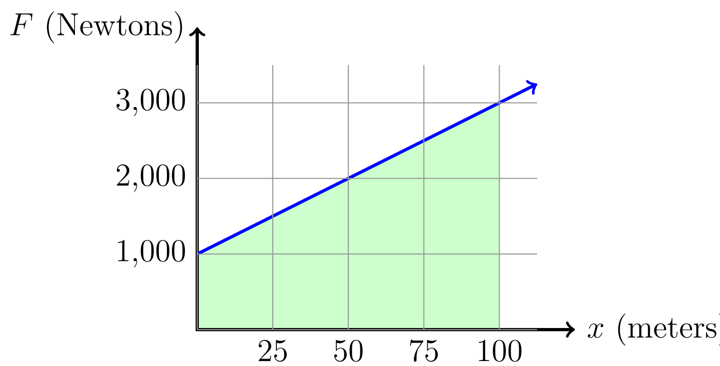
(a) $\sum_{n=0}^{\infty} \frac{(-1)^n 5^{n+1}}{6^n}$

(b) $\sum_{k=2}^{\infty} \frac{\ln k}{k-1}$

(c) $\sum_{n=1}^{\infty} \cos(n\pi)$

5. Find all values of x for which the Taylor series $\sum_{n=0}^{\infty} \frac{2^n}{n} x^n$ converges.

6. Suppose I am pushing a heavy object over snow covered ground. The further I go, the deeper the snow gets, making me use more and more force to push the object. If the force I use as I push the object 100 meters is shown in the graph below, find the amount of work I did.

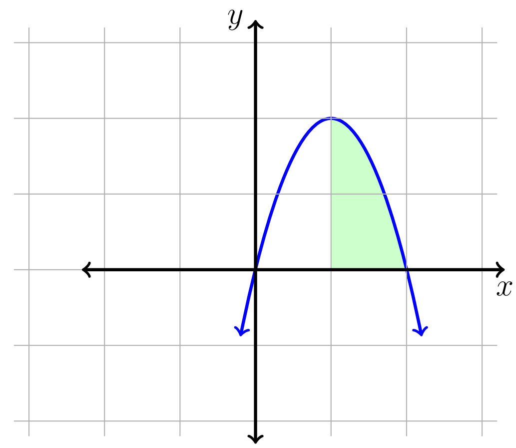


7. Find the following limits.

(a) $\lim_{x \to 0} \frac{\cos 2x - 1}{x^2}$

(b) $\lim_{x \to \infty} \frac{e^x + \ln x}{x^2 + 100}$

8. Let $\mathcal{R}$ be the region under the curve $y = 4x - 2x^2$ from $x = 1$ to $x = 2$.



(a) Find the volume of the solid formed by revolving $\mathcal{R}$ around the y-axis.

(b) Set up, but do not evaluate, an integral for the volume of the solid formed by revolving $\mathcal{R}$ around the x-axis.

9. Suppose that $f(x) = \sin(x^3)$.

(a) Find a Maclaurin series for $f(x)$.

(b) Use part (a) to find an infinite series for the integral $\int_0^1 \sin(x^3)\,dx$.

10. Evaluate the following integrals.

(a) $\int x^4 \ln x \, dx$

(b) $\int \frac{x^3 + 4}{x^2 - 4} \, dx$

11. Solve the following logarithm problems.

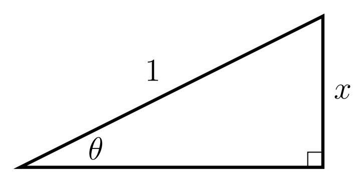
(a) Simplify $\log_5(50) + \log_5(\frac{5}{2})$.

(b) Solve the equation $2^{x-1} = e^5$.

12. Use logarithmic differentiation to find the derivative of $y = (1+x)^x$.

13. Use the trig substitution $x = \sin\theta$ to evaluate

$$\int x^3\sqrt{1-x^2}\,dx$$



14. Simplify $\tan(\arcsin(x^2))$ using a reference triangle.

15. Find the area between the two curves $f(x) = x^2 - 6x$ and $g(x) = 3 - 4x$.

16. Estimate the worst case error in using the second degree Maclaurin polynomial $1 - \dfrac{x^2}{2}$ to approximate $\cos(0.3)$.

17. Find the sums of the following geometric series.

(a) $7 + 1 + \frac{1}{7} + \frac{1}{49} + \ldots$

(b) $x^2 + \dfrac{x^3}{5} + \dfrac{x^4}{25} + \dfrac{x^5}{125} + \ldots$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-3)^n}{4^{n-1}}$

18. The slope field below corresponds to the differential equation $y' = -\frac{1}{4}x(y+2)$. What does the solution of the differential equation with initial condition $y(-2) = 0$ look like? Draw a rough sketch of the solution on the slope field below. You do not need to solve the differential equation.