Calculus II - Math 142 **Final Exam Review Solutions**

1. Evaluate the following integrals.

   (a) $\displaystyle\int e^x \cos(e^x)\,dx.$

   Solution: (Hint) Use the u-substitution $u = e^x$.

   (b) $\displaystyle\int \tan^5\theta \sec^3\theta\,d\theta.$

   Solution: (Hint) Keep a factor of $\sec\theta\tan\theta\,d\theta$ as your integrating factor, and covert the other four tangents to secants using the identity $\sec^2\theta - 1 = \tan^2\theta$.

   (c) $\displaystyle\int x^2\cos(3x)\,dx$

   Solution: (Hint) Use the tabular method.

   $$\tfrac{1}{3}x^2\sin 3x + \tfrac{2}{9}\cos 3x - \tfrac{2}{27}\sin 3x + C$$

2. Find the third degree Taylor polynomial for $f(x) = x^3 + 2x - 3$ centered at $c = 2$.

   Solution: (Hint) Make a table of derivatives.

   $$P_3(x) = 9 + 14(x-2) + \frac{12}{2!}(x-2)^2 + \frac{6}{3!}(x-2)^3$$

3. Solve the differential equation $\frac{dy}{dx} = \frac{\cos x}{y^2}$ with initial condition $y(\pi) = 2$.

Solution:

$$y = \sqrt[3]{3 \sin x + 8}$$

4. For each of the following series, determine whether it converges or diverges and give your reasoning.

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n 5^{n+1}}{6^n}$

Solution: (Hint) This is a geometric series, so you can tell whether it converges by finding the common ratio. It is also an alternating series, so you could also use the alternating series test.

(b) $\sum_{k=2}^{\infty} \frac{\ln k}{k-1}$

Solution: Diverges by comparison with the harmonic series $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots$ which is a p-series with $p = 1$ (so it diverges).

(c) $\sum_{n=1}^{\infty} \cos(n\pi)$

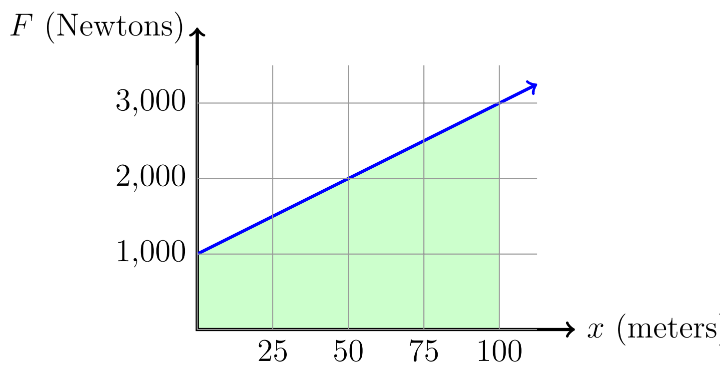
Solution: The terms of this series alternate between $+1$ and -1. Since the terms don't converge to zero, the series cannot converge.

5. Find all values of x for which the Taylor series $\sum_{n=0}^{\infty} \frac{2^n}{n} x^n$ converges.

Solution:

$$[-\tfrac{1}{2}, \tfrac{1}{2})$$

6. Suppose I am pushing a heavy object over snow covered ground. The further I go, the deeper the snow gets, making me use more and more force to push the object. If the force I use as I push the object 100 meters is shown in the graph below, find the amount of work I did.



Solution: (Hint) Work is $\int F\,dx$ which is just the area under this curve.

200,000 Newton-meters (Joules)

7. Find the following limits.

(a) $\lim_{x \to 0} \frac{\cos 2x - 1}{x^2}$

Solution:

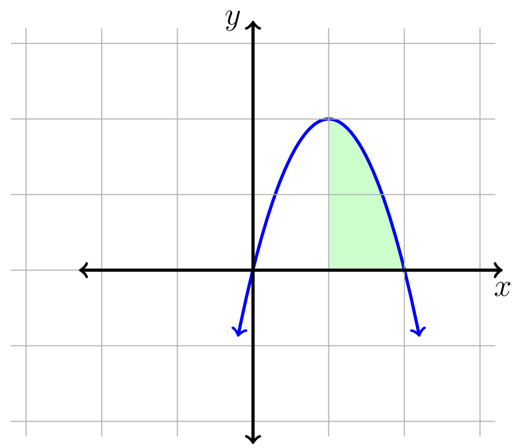
$$-2$$

(b) $\lim_{x \to \infty} \frac{e^x + \ln x}{x^2 + 100}$

Solution:

$$\infty$$

8. Let $\mathcal{R}$ be the region under the curve $y = 4x - 2x^2$ from $x = 1$ to $x = 2$.



(a) Find the volume of the solid formed by revolving $\mathcal{R}$ around the y-axis.

Solution:

$$V = \frac{11\pi}{3}$$

(b) Set up, but do not evaluate, an integral for the volume of the solid formed by revolving $\mathcal{R}$ around the x-axis.

Solution:

$$V = \int_1^2 \pi(4x - 2x^2)^2\,dx$$

9. Suppose that $f(x) = \sin(x^3)$.

(a) Find a Maclaurin series for $f(x)$.

Solution:

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+3}}{(2n+1)!} \quad \text{or} \quad x^3 - \frac{x^9}{3!} + \frac{x^{15}}{5!} - \frac{x^{21}}{7!} + \dots$$

(b) Use part (a) to find an infinite series for the integral $\displaystyle\int_0^1 \sin(x^3)\,dx$.

Solution:

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(6n+4)(2n+1)!} \quad \text{or} \quad \frac{1}{4} - \frac{1}{10\cdot 3!} + \frac{1}{16\cdot 5!} - \frac{1}{22\cdot 7!} + \dots$$

10. Evaluate the following integrals.

(a) $\displaystyle\int x^4 \ln x \, dx$

Solution: (Hint) The tabular method won't work since $\ln x$ isn't easy to integrate. Use integration by parts instead, with $u = \ln x$ and $dv = x^4 \, dx$.

$$\tfrac{1}{5}x^5 \ln x - \tfrac{1}{25}x^5 + C$$

(b) $\displaystyle\int \frac{x^3+4}{x^2-4} \, dx$

Solution: (Hint) Use polynomial long-division first, then partial fractions.

$$\frac{x^2}{2} + 3\ln|x-2| + \ln|x+2| + C$$

11. Solve the following logarithm problems.

(a) Simplify $\log_5(50) + \log_5(\frac{5}{2})$.

Solution:

$$3$$

(b) Solve the equation $2^{x-1} = e^5$.

Solution:

$$x = \frac{5}{\ln 2} + 1$$

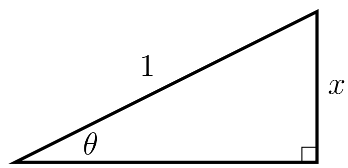
12. Use logarithmic differentiation to find the derivative of $y = (1+x)^x$.

Solution:

$$y' = \left(\frac{x}{1+x} + \ln(1+x)\right)(1+x)^x$$

13. Use the trig substitution $x = \sin\theta$ to evaluate

$$\int x^3\sqrt{1-x^2}\,dx$$

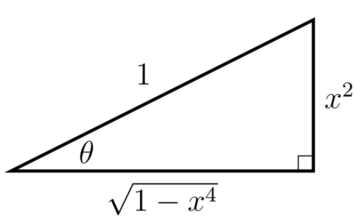


Solution:

$$\frac{(1-x^2)^{5/2}}{5} - \frac{(1-x^2)^{3/2}}{3} + C$$

14. Simplify $\tan(\arcsin(x^2))$ using a reference triangle.

Solution: (Hint) The right reference triangle is:



15. Find the area between the two curves $f(x) = x^2 - 6x$ and $g(x) = 3 - 4x$.

Solution: (Hint) You have to figure out where the two functions intersect first.

$$\frac{32}{3}$$

16. Estimate the worst case error in using the second degree Maclaurin polynomial $1 - \dfrac{x^2}{2}$ to approximate $\cos(0.3)$.

Solution: (Hint) Use the alternating series error formula.

$$\text{Error} < \frac{0.3^4}{4!}$$

17. Find the sums of the following geometric series.

(a) $7 + 1 + \frac{1}{7} + \frac{1}{49} + \ldots$

Solution:

$$\frac{49}{6}$$

(b) $x^2 + \dfrac{x^3}{5} + \dfrac{x^4}{25} + \dfrac{x^5}{125} + \ldots$

Solution:

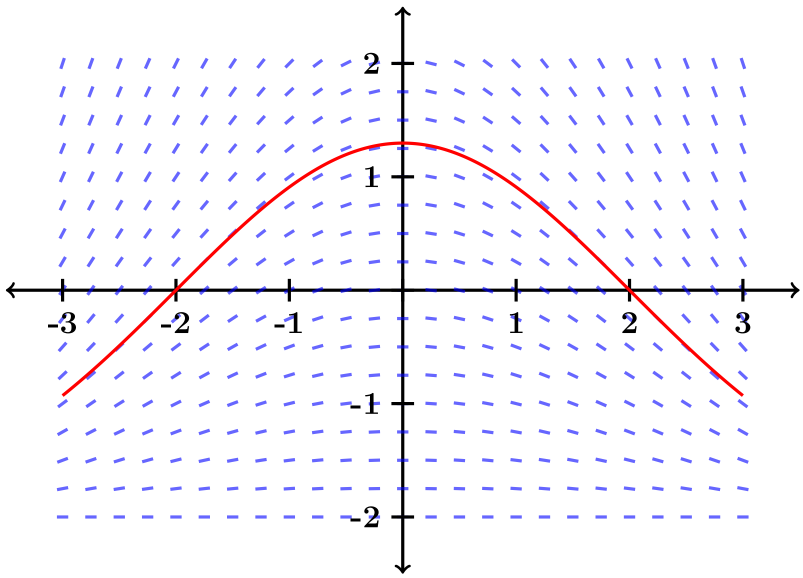
$$\frac{x^2}{1 - \frac{x}{5}}$$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-3)^n}{4^{n-1}}$

Solution:

$$= \frac{4}{1 - \frac{-3}{4}} = \frac{16}{7}$$

18. The slope field below corresponds to the differential equation $y' = -\frac{1}{4}x(y+2)$. What does the solution of the differential equation with initial condition $y(-2) = 0$ look like? Draw a rough sketch of the solution on the slope field below. You do not need to solve the differential equation.



Solution: The red curve follows the slope field and passes through the point $(-2, 0)$, so it is the graph of the solution of the differential equation.