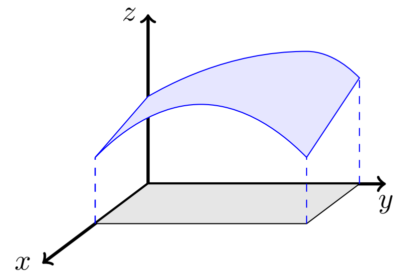
### Key Concepts




- The graph of a function of two variables $z = f(x, y)$ is a surface.
- $\iint_R f(x, y)\,dx\,dy$ is the volume under the surface over the region $R$.

1. Let $f(x, y) = \frac{1}{3}(x + y)$ on the rectangle $0 \le x \le 2$, $0 \le y \le 1$.
   (a) Show that $f(x, y)$ is a valid joint density function.

   (b) If $X, Y$ are random variables with this joint density function, find $P(X \ge 1)$.

   (c) Set up an integral to find $P(X \ge Y)$. Which option will be easier and why?

   A. A type I integral $\displaystyle\int_a^b \int_{f_1(x)}^{f_2(x)} f(x, y)\,dy\,dx$,

   B. A type II integral: $\displaystyle\int_c^d \int_{g_1(y)}^{g_2(y)} f(x, y)\,dx\,dy$?

2. James Bond is trapped in a room with a bomb. The bomb will explode when a certain radioactive trigger decays. The time until the trigger decays has an exponential distribution with parameter $\lambda = 2$ per hour. James Bond must pick a lock to escape the room. Suppose the time it takes him to pick the lock is also exponentially distributed with parameter $\lambda = 3$ per hour. Find the probability that James Bond is able to escape the room before the bomb explodes. Hint: *the two random variables are independent here, so the joint distribution function is the product of their individual probability density functions. That is:*

$$f(x, y) = (2e^{-2x})(3e^{-3y}) = 6e^{-2x-3y}.$$