

Homework 1 - Math 421

Name: _____

Due Friday, August 25. Be sure to show any work you needed to do. You can use a calculator or computer, but give exact (not decimal) answers when possible.

1. In the game Wordle, you guess the letters for a 5 letter word. If you pick a random string of any five letters (repeat letters allowed), what is the probability you will get a word? The original Wordle game recognized 10665 words.
2. Only 6762 of the words that Wordle recognizes have no repeat letters. What if you randomly select a string with no repeated letters. What is the probability that your string is a recognizable word?
3. In a round-robin tournament with n players, every player plays every other player exactly once.
 - (a) How many games total are played in a round-robin tournament with n players?
 - (b) How many different outcomes for the tournament are possible?
4. Three people get on an elevator going up in a 10 story building. Each presses the button for a different floor from 2 to 10. What is the probability that the buttons for 3 consecutive floors are pressed?

5. Write a computer program with a function to verify that the formula

$$\sum_{k=3}^{n+1} \binom{k-1}{2} \binom{n+3-k}{2} = \binom{n+3}{5}$$

is true for any integer $n > 1$. *You can use any programming language (Python, C++, R, etc.). Either e-mail me your code or, if it is short enough, you can copy your code here. I recommend using built in functions to compute the binomial coefficients.*

6. Give a story proof that

$$\sum_{k=3}^{n+1} \binom{k-1}{2} \binom{n+3-k}{2} = \binom{n+3}{5}.$$

Hint: Think about the middle element in a set of size 5 chosen from $\{1, \dots, n+3\}$.