

Homework 7 - Math 421

Name: _____

Due Friday, October 13. Be sure to show any work you needed to do. You can use a calculator or computer, but give exact (not decimal) answers when possible.

1. A gas station is supplied with gasoline once a week. The probability density function for how much gas it sells each week (in thousands of gallons) is

$$f(x) = 30(x^4 - x^5), \text{ with support } 0 \leq x \leq 1.$$

- (a) Find the CDF for this distribution by integrating the PDF.
- (b) Find the expected value and variance for this distribution.
- (c) How much gas would this gas station sell during a week where demand was in the 90th percentile? Hint: Don't invert the CDF, that's too hard. Instead, graph the CDF in Desmos to figure out the answer. Where does the graph cross the 90th percentile?
2. Suppose that X is a random variable and a, b are constants. Recall that the standard deviation of X is $\text{SD}(X) = \sqrt{\text{Var}(X)}$. Find $\text{SD}(aX + b)$.
3. Suppose the time that it takes Bob to finish his homework each night has an exponential distribution with rate $\lambda = 0.5/\text{hour}$.
- (a) Find the probability that the homework takes over 2 hours.
- (b) If Bob is still working after 2 hours, find the conditional probability that his homework will take over 3 hours that night.

4. The CDF for an $\text{Exp}(\lambda)$ random variable X is $F(x) = 1 - e^{-\lambda x}$ for all $x \geq 0$. Find the formula for the inverse CDF function $F^{-1}(p)$.

5. Annual rainfall in Farmville, VA is normally distributed with mean 44 inches and standard deviation 7 inches. Assuming that rainfall amounts are independent from one year to the next (which is a dubious assumption!), what is the probability that it will take more than 10 years before there is a drought year with less than 30 inches of rain?

6. SAT scores (out of 1600) are distributed normally with a mean of 1100 and a standard deviation of 200. Suppose a school council awards a certificate of excellence to all students who score at least 1350 on the SAT, and suppose we pick one of the recognized students at random. What is the probability this student's score will be at least 1500?

7. Write down (but don't evaluate) an integral that represents the kurtosis of a normal distribution with mean 70 and standard deviation 3.

8. Write down (but don't evaluate) an infinite series that represents the 3rd moment of a Geometric random variable with parameter p .