Calculus I — Math 161 Final Exam

Name: _____

Instructions: You must show all work to earn full credit. No calculators allowed. If you do not have room in the given space to answer a question, use the back of the formula sheet and *indicate clearly* which work goes with which problem.

Problem	Maximum Points	Your Points
1	10	
2	8	
3	6	
4	6	
5	16	
6	8	
7	8	
8	6	
9	6	
10	10	
11	8	
12	12	
13	8	
14	8	
Total	120	

1. (10 points)

   (a) Find the equation of the tangent line for the curve $y = (x^2 + 1)(1 - x^3)$ at the point $x = 1$.

   (b) Find y' if $\sin(y + 1) = xy$.

2. (8 points) A cylindrical container with no top is to be constructed to hold 6000 cm^3 of a liquid. Suppose that the cost of the material used for the bottom is 5 cents/cm^2, and the cost of the material used for the curved lateral side is 3 cents/cm^2.

   (a) Find an expression for the the cost of the container in terms of the radius r and the height of the container h.

   (b) What is the radius of the least expensive container matching the description above?

3. (6 points) Prove that $-3x^4 - 2x^3 + x^2 + 1 = 0$ has a solution on the interval $[0, 1]$.

4. (6 points) The conclusion of the Mean Value Theorem tells us that there is a c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Explain why that conclusion does not apply to the function $f(x) = |3x - 1|$ on the interval $[-1, 1]$.

5. (16 points) Calculate the following integrals.

(a) $\displaystyle\int \cos\theta(\sin\theta)^{-1/2}\,d\theta$

(b) $\displaystyle\int_0^1 x^2(1+x^3)^4\,dx$

(c) $\displaystyle\frac{d}{dx}\int_x^5 \cos^3(2t)\,dt$

(d) Find the area under the curve $y = x^2$ from $x = -1$ to $x = 2$.

6. (8 points)

(a) Let $T(t)$ represent the temperature in Carlisle, PA (in °F) as a function of the time t hours after midnight. Explain in words what is the meaning of $T'(6)$ and also explain what units are used to measure $T'(t)$.

(b) If water leaks from a tank at a rate of $r(t)$ gallons per minute at time t minutes, what does $\int_0^{180} r(t)\,dt$ represent?

7. (8 points) Suppose a rock is thrown upwards from the edge of a cliff with a velocity of 16 ft/s. The acceleration of gravity is -32 ft/s^2.

   (a) If the rock hits the ground at $t = 5$ seconds, how tall is the cliff?

   (b) What is the total distance traveled by the rock?

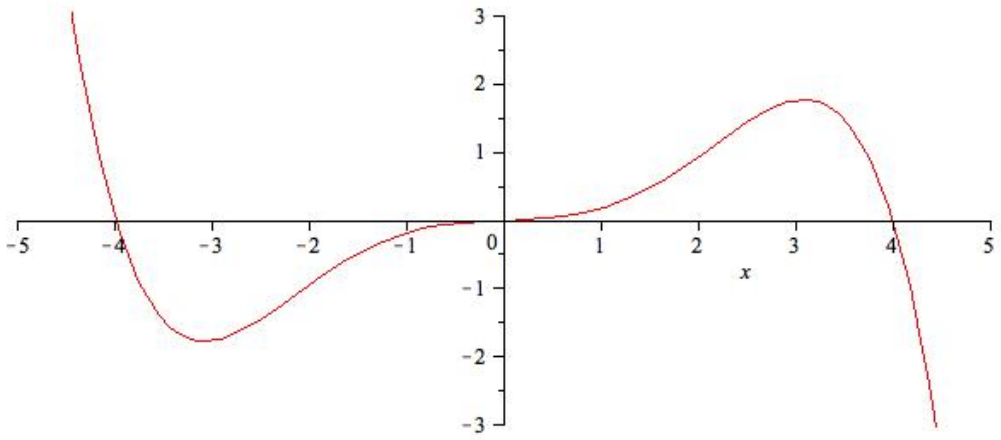
8. (6 points) Use the definition of the derivative to calculate:

$$\frac{d}{dx}\frac{1}{1-x}$$

9. (6 points) Estimate $\frac{1}{5.25}$ using a linear approximation.

10. (10 points) A 5 meter ladder is leaning against a vertical wall. Suppose that the top of the ladder begins sliding down the wall while the bottom of the ladder slides away from the wall. How fast is the top of the ladder decending when the bottom of the ladder is 3 meters from the wall and the bottom is sliding at a rate of 2 m/s?

11. (8 points) The graph below represents $f(x)$.



(a) Sketch a graph of $f'(x)$. Make sure you include any important features (x,y-intercepts, local max/min, etc.).

(b) Sketch a rough graph of $g(x) = \int_0^x f(t)\,dt$ for $0 \leq x \leq 5$.

12. (12 points)

(a) $\lim_{x \to \infty} \dfrac{3x^3 + 2x + 1}{4x - x^3}$

(b) $\lim_{x \to 1} \dfrac{x^7 - 1}{x - 1}$

(c) $\lim_{x \to 0^-} \dfrac{|x|}{2x}$

13. (8 points) Consider the function

$$f(x) = \begin{cases} Ax + 3 & \text{if } x < 1 \\ 5 & \text{if } x = 1 \\ x^2 + B & \text{if } x > 1 \end{cases}$$

(a) Find all values of A and B such that $f(x)$ is continuous for all x.

(b) Are there values of A and B such that $f(x)$ is differentiable for all x?

14. (8 points) Find the following derivatives. Do not simplify your answers.

(a) $\dfrac{d}{dx}\, x \cos 5x$

(b) $\dfrac{d}{dx} \sqrt{\dfrac{2x^2 - 1}{3x^2 + 2}}$

MATH 161 - Midterm 1 Formula Sheet

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Common Trigonometric Ratios

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\sin\theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1

Obscure Trigonometry Ratios

$$\cot\theta = \frac{\cos\theta}{\sin\theta}, \qquad \csc\theta = \frac{1}{\sin\theta}$$

Angle Addition Formulas

$$\sin\alpha \pm \beta = \sin\alpha\cos\beta \pm \sin\beta\cos\alpha$$

$$\cos\alpha \pm \beta = \cos\alpha\cos\beta \mp \sin\alpha\cos\beta$$

Linear Approximation Formulas

$$L(x) = f(a) + f'(a)(x - a) \qquad \frac{\Delta y}{\Delta x} \approx \frac{dy}{dx}$$

Riemann Sum Formula

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta_i$$

Volume & Area Formulas

Volume of a Sphere: $V = \frac{4}{3}\pi r^3$

Surface Area of a Sphere: $S = 4\pi r^2$

Volume of a Cone: $V = \frac{1}{3}\pi r^2 h$