Math 142 Group Projects

For this project you must work in groups of at most 3 students. You and your group will have to pick a topic by Friday, April 10. No two groups may present the same topic and the topics are awarded on a first come first serve basis. Each group will have 10-12 minutes to present their topic during the last week of class.

Leonhard Euler and Euler's formula

1. Give biographical information about Euler such has where he lived, when he was born, what his background was, etc.
2. Briefly describe some of his most famous works.
3. Explain Euler's formula and show how it can be derived using the Taylor series for sine, cosine, and e^x.
4. Show that Euler's formula immediately implies Euler's identity $e^{i\pi} + 1 = 0$ which relates the five most important numbers in all of mathematics.

Archimedes and the quadrature of the parabola

1. Begin by giving biographical information about Archimedes. Describe what is known about him, where he was born, when he lived, etc.
2. Give a description of how Archimedes was able to compute the area inside of a parabola without using integral calculus.
3. Use integral calculus to find the same area that Archimedes found.

Gabriel's horn Gabriel's horn is a shape that has finite volume but infinite surface area.

1. Read the section in the book about calculating surface area using integrals. As part of your presentation, you must introduce the formula for surface area to the class.
2. Describe Gabriel's horn. In your presentation, calculate both the surface area and the volume of Gabriel's horn.
3. Explain why Gabriel's horn is considered paradoxical.

Buffon's needle This is a way to estimate π by tossing a needle on a floor between two straight lines. For this project, you will need to read about Buffon's needle and explain how you would use it to approximate π. You may want to use an applet such as the one at

http://www.math.uah.edu/stat/applets/BuffonNeedleExperiment.html

to demonstrate how Buffon's needle works.

The gamma function At first glance it appears that there is no continuous analog to the factorial function $n!$. In fact there is. It is a famous function called the *gamma function.* The gamma function has the form $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}\,dt$.

1. Find $\Gamma(1)$, $\Gamma(2)$ and $\Gamma(3)$.
2. Use integration by parts to show that $\Gamma(x) = (x-1)\Gamma(x-1)$
3. Prove that $\Gamma(n) = (n-1)!$ by using mathematical induction.

e is irrational Our goal is to show that e is an irrational number. To do so we use a proof technique called a *proof by contradiction*. We assume that e is rational and show that this leads to a contradiction. Thus we conclude that e is irrational.

1. If e were rational, then it would be a fraction $e = \frac{m}{n}$ where both m and n are integers and $n \geq 2$. Use Taylor's formula to write

$$\frac{m}{n} = e = 1 + \frac{1}{1!} + \frac{1}{2!} + ... + \frac{1}{n!} + \frac{e^z}{(n+1)!} = S_n + R_n$$

   where

$$S_n = 1 + \frac{1}{1!} + \frac{1}{2!} + ... + \frac{1}{n!}$$

   is the n^{th} partial sum for e and

$$R_n = \frac{e^z}{(n+1)!}$$

   is the remainder (see Taylor's theorem).

2. Show that $n!(\frac{m}{n} - S_n)$ is an integer.
3. Show that $0 < n!(R_n) < 1$.
4. Show that $\frac{m}{n} - S_n = R_n$
5. Explain why this is a contradiction. What does this contradiction force us to conclude?