Calculus II — Math 142 Final Exam

Name: ____________________

Instructions: You must show all work to earn full credit. No calculators allowed. If you do not have room in the given space to answer a question, use the back of the formula sheet and *indicate clearly* which work goes with which problem.

Problem	Maximum Points	Your Points
1	8	
2	8	
3	6	
4	8	
5	8	
6	8	
7	8	
8	8	
9	8	
10	8	
11	8	
12	8	
13	6	
Total	100	

1. (8 points) Evaluate the following integrals.

   (a) $\displaystyle\int \cos^2\theta \sin\theta \, d\theta$

   (b) $\displaystyle\int \frac{3}{x(x-1)} \, dx$

2. (8 points)

   (a) Find a Maclaurin series for $\dfrac{x}{1-x^2}$.

   (b) Find the first 4 nonzero terms in the Maclaurin series for $\ln(1-x^2)$. Hint: you can use the fact that $\ln(1-x^2) = -2\displaystyle\int \frac{x}{1-x^2}\,dx$.

3. (6 points) Find the interval of convergence for the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(x+1)^n}{3^n}$.

4. (8 points) Find the following limits.

(a) $\displaystyle\lim_{x\to 0} \frac{\sin(x)}{\tan(2x)}$

(b) $\displaystyle\lim_{x\to 0} \frac{\ln(1+3x)}{x}$

(c) $\displaystyle\lim_{n\to\infty} (2^n)(3^{-n})$

5. (8 points) Let $\mathcal{R}$ denote the region under the curve $y = 4 - x^2$ from $x = 0$ to $x = 2$. Find the volume obtained by revolving $\mathcal{R}$ around the y-axis.

6. (8 points)

(a) Find the inverse of the function $f(x) = \sqrt{5^x + 1}$.

(b) Suppose that $T(t)$ is the function that gives the current temperature in Hampden-Sydney, VA where t is time measured in days since January 1st. Is $T(t)$ an invertible function? Explain why or why not.

7. (8 points) Find the sums of the following series.

(a) $\sum_{n=0}^{\infty}(2^n)(5^{-n})$

(b) Use a Maclaurin series to find the sum of the series $1 - \frac{(\pi)^2}{2!} + \frac{(\pi)^4}{4!} - \frac{(\pi)^6}{6!} + ...$

8. (8 points) Evaluate the following integrals.

(a) $\int_0^1 t\, e^{2t}\, dt$

(b) $\int_0^{\sqrt{\pi}/2} x \sec^2(x^2)\, dx$

9. (8 points)

(a) Find the general solution to the differential equation

$$\frac{dy}{dx} = \frac{2x}{e^y}$$

(b) For the equation above, find the solution that satisfies the initial condition

$$y(0) = 0.$$

10. (8 points) For each of the follow series, state whether it converges, diverges, or there is not enough information. Be sure to give your reasoning in each case.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2 \ln k}$

(b) $\displaystyle\sum_{n=0}^{\infty} n^{-4/3}$

(c) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{n}{n+1}$

11. (8 points) Use the trig substitution $x = \sin\theta$ to find the integral $\displaystyle\int \frac{1}{(1-x^2)^{3/2}}\,dx$.

12. (8 points) Find the area between the curves $y = 4 - x^2$ and $y = x + 2$.

13. (6 points) Use logarithmic differentiation to find the derivative of

$$y = x^{\sin(x)}$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Common Trigonometric Ratios

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\sin\theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1

Arc Length Formula

$$L = \int_a^b \sqrt{1 + (y')^2}\, dx$$

Obscure Trigonometry Ratios

$$\cot\theta = \frac{\cos\theta}{\sin\theta}, \qquad \csc\theta = \frac{1}{\sin\theta}$$

Trigonometry Identities

$$\sin 2x = 2\sin x\cos x, \qquad \cos 2x = \cos^2 x - \sin^2 x$$

$$\sin^2 x = \frac{1-\cos 2x}{2}, \qquad \cos^2 x = \frac{1+\cos 2x}{2}$$

Selected Derivatives

$$\frac{d}{dx}\tan x = \sec^2 x, \qquad \frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x\tan x, \qquad \frac{d}{dx}\csc x = -\csc x\cot x$$

$$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}, \qquad \frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\sec^{-1} x = \frac{1}{x\sqrt{x^2-1}}, \qquad \frac{d}{dx}a^x = a^x \ln a$$

Selected Integrals

$$\int a^u\,du = \left(\frac{1}{\ln a}\right) a^u + C \qquad \int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\frac{u}{a} + C$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\arctan\frac{u}{a} + C \qquad \int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$$

Taylor Polynomial of Degree n

$$P_n(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + ... + \frac{f^{(n)}(a)}{n!}(x-a)^n$$