Complex Analysis Homework #1
Due Monday, January 26

(1) Draw the set of points z in the complex plane that form solutions to each of the following equations.
(a) $|z| = 3$
(b) $|z - 1| = 2$
(c) $|z - 3| = |z + 3|$
(d) $\mathrm{Re}\,[(1 - i)z] = 0$
(e) $|z^2 - 2z + 1| = 0$.

(2) Show that $|z| = 1$ if and only if $\bar{z} = 1/z$.

(3) Prove the identity

$$1 + z + z^2 + ... + z^n = \frac{1 - z^{n+1}}{1 - z}.$$

Hint: You can use induction.

(4) Give the polar representation for each of the following complex numbers.
(a) $-\sqrt{3} - i$
(b) $(2 - i)^2$
(c) $[(1 + i)/\sqrt{2}]^4$

(5) Find all solutions to the equations below.
(a) $(z + 1)^4 = 2 + 2i$
(b) $z^3 = 8$.