Complex Analysis Homework #4
Due Friday, February 27

1. Evaluate the following line integrals by two methods: (i) directly and (ii) using Green's theorem.

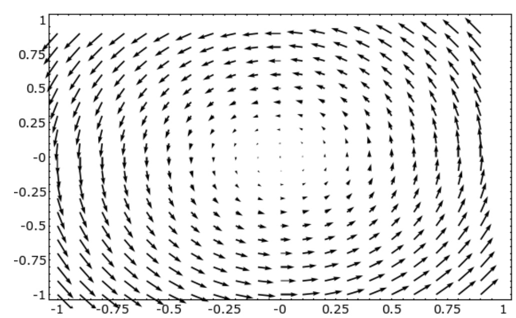
   (a) $\oint_C x^2 y\, dx + xy^3\, dy$
   C is the square with vertices $(0,0), (0,1), (1,0)$, and $(1,1)$.

   (b) $\oint_C (x+2y)\, dx + (x-2y)\, dy$
   C consists of the arc of the parabola $y = x^2$ from $(0,0)$ to $(1,1)$ followed by the line segment from $(1,1)$ back to $(0,0)$.

2. For each of the following vector fields, determine if the field is conservative or not. If the field is conservative, find the potential function $f(x,y)$.

   (a) $F(x,y) = (3x^2 - 4y)\mathbf{i} + (4y^2 - 2x)\mathbf{j}$
   (b) $F(x,y) = (x^2 + y)\mathbf{i} + (y^2 + x)\mathbf{j}$
   (c) $F(x,y) = (ye^x + \sin y)\mathbf{i} + (e^x + x\cos y)\mathbf{j}$

3. Consider the vector field F shown below.



   (a) Suppose that C is the upper-half of the unit circle parametrized by $\gamma = (\cos t, \sin t)$. If you integrate the line integral $\int_C F \cdot d\gamma$ from $t = 0$ to $t = \pi$, will you get a positive or negative value?
   (b) What if C is the lower-half of the unit circle parametrized by $\gamma = (\cos t, -\sin t)$ from $t = 0$ to $t = \pi$. Is $\int_C F \cdot d\gamma$ positive or negative? Explain why.
   (c) Is $\int_C F \cdot d\gamma$ path independent? Is F conservative? Explain.

4. Prove that if C is a simple, smooth, closed curve in $\mathbb{R}^2$, then $\oint_C x\, dy$ is the area of the region enclosed by C. (Hint: Recall that $\iint_D 1\, dA$ is the area of D.)

5. Calculate the following complex line integrals.

(a) $\int_C |z|^2 \, dz$
C is the line segment from 1 to i.

(b) $\int_C |z| + 1 \, dz$
C is the quarter arc of the unit circle from 1 to i.