Complex Analysis Homework #8
Due Monday, April 20th

1. Let f and g be analytic on a domain containing a simple closed curve γ and its inside. Show that if $|f(z)| > |g(z)|$ for all $z \in \gamma$, then the equation $f(z) = g(z)$ has the same number of solutions inside γ as $f(z) = 0$ (counting multiplicity).

2. Use the Rouche's theorem to find the number of zeroes of $z^3 - 3z + 1$ inside the annulus $1 < |z| < 2$.

3. The function $f(z) = \begin{cases} (z-1)^{-1} & |z| < 1, \\ 1 & |z| \geq 1 \end{cases}$ is analytic in the open unit disc D, and $|f(z)| = 1$ everywhere on the boundary of D, however $|f(z)|$ does not attain its maximum on the boundary. Explain why this does not contradict the Maximum Modulus Principle.

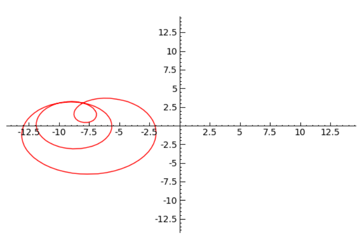
4. Use the Argument Principle to match each of the following functions with the corresponding image of the unit circle $|z| = 1$. Explain your reasoning for each.

   (a) $\dfrac{z^2}{z+2}$

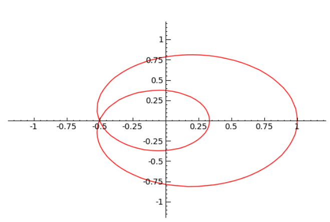
   (b) $4z^3 - 2iz$

   (c) $3z^3 - 2z^2 + 2iz - 8$

I.



II.



III.