Calculus II - Math 142 **Final Exam**

Name: _____

Instructions: You must show all work to earn full credit. No calculators allowed. If you do not have room in the given space to answer a question, use the back of the formula sheet and *indicate clearly* which work goes with which problem.

Problem	Maximum Points	Your Points
1	8	
2	8	
3	8	
4	8	
5	4	
6	4	
7	8	
8	8	
9	8	
10	8	
11	8	
12	4	
13	8	
14	8	
Total	100	

1. (8 points) Evaluate the following integrals.

   (a) $\int e^x \cos(e^x)\, dx.$

   (b) $\int \tan^5 \theta \sec^2 \theta\, d\theta.$

2. (8 points) Find the third degree Taylor polynomial for $f(x) = x^3 + 2x - 3$ centered at $c = 2$.

3. (8 points) Solve the differential equation.

$$\frac{dy}{dx} = \frac{\cos x}{y^2}$$

4. (8 points) For each of the following series, state whether it converges or diverges and give your reasoning.

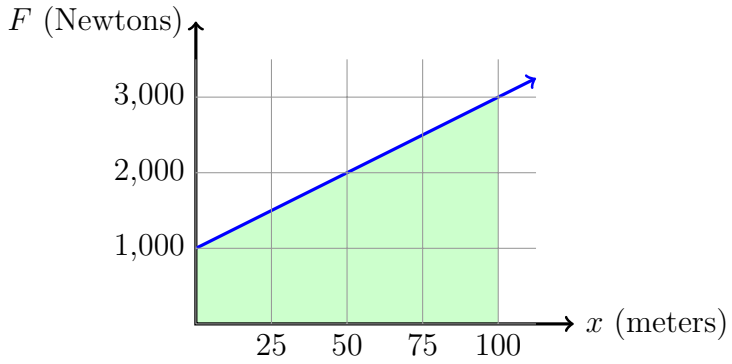
(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 5^{n+1}}{6^n}$

(b) $\displaystyle\sum_{k=2}^{\infty} \frac{\ln k}{k-1}$

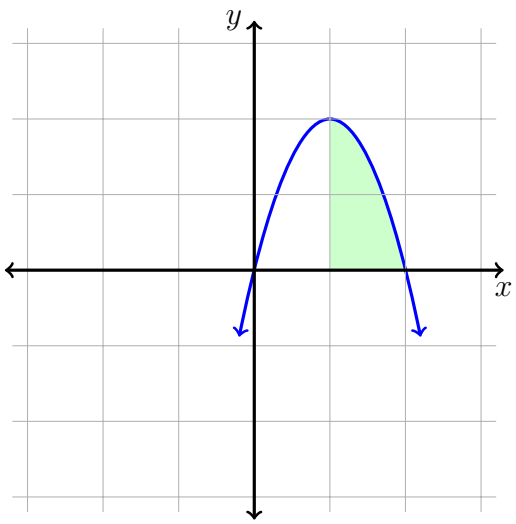
(c) $\displaystyle\sum_{n=1}^{\infty} \cos(n\pi)$

5. (4 points) Is the function $y = x^2 - 4x + 5$ invertible? If so, find the inverse. If not, explain why not.

6. (4 points) Suppose I am pushing a heavy object over snow covered ground. The further I go, the deeper the snow gets, making me use more and more force to push the object. If the force I use as I push the object 100 meters is shown in the graph below, find the amount of work I did.



7. (8 points) Let $\mathcal{R}$ be the region under the curve $y = 4x - 2x^2$ from $x = 1$ to $x = 2$.



(a) Find the volume of the solid formed by revolving $\mathcal{R}$ around the y-axis.

(b) Setup, but do not evaluate an integral for the volume of the solid formed by revolving $\mathcal{R}$ around the x-axis.

8. (8 points) Suppose that $f(x) = \sin(x^3)$.

   (a) Find a Maclaurin series for $f(x)$.

   (b) Use part (a) to find an infinite series for the integral $\int_0^1 \sin(x^3)\,dx$.

9. (8 points) Evaluate the following integrals.

(a) $\int x^4 \ln x \, dx$

(b) $\int \frac{x^3 + 4}{x^2 - 4} \, dx$

10. (8 points) Solve the following logarithm problems.

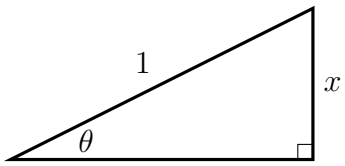
(a) Simplify $\log_5(50) + \log_5(\frac{5}{2})$.

(b) Solve the equation $2^{x-1} = e^5$.

(c) Use logarithmic differentiation to find the derivative of $y = (1 + x)^x$.

11. (8 points) Use the trig substitution $x = \sin\theta$ to evaluate

$$\int x^3\sqrt{1-x^2}\,dx$$



12. (4 points) Find each of the infinite sums below using a type of series you know.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,2^n}$

(b) $2 - \frac{1}{2} + \frac{1}{8} - \frac{1}{32} + \ldots$

13. (8 points) I know that the function $f(x) = \dfrac{1}{(x-2)^2}$ is always positive.

Therefore I expected to get a positive answer when I tried to find the area under this curve from $x = 1$ to $x = 3$. However, I got -2 as my answer. Explain what is wrong with my calculation (it may help to draw a picture of $f(x)$).

$$\int_1^3 \frac{1}{(x-2)^2}\,dx = \int_1^3 (x-2)^{-2}\,dx = \left[-(x-2)^{-1}\right]_1^3 = -1 - 1 = -2$$

What is the actual value of $\displaystyle\int_1^3 \frac{1}{(x-2)^2}\,dx$? Be sure to show your calculations.

14. (8 points) Find the area between the two curves $f(x) = x^2 - 6x$ and $g(x) = 3 - 4x$.

(Extra Credit) Estimate the worst case error in using the second degree Taylor polynomial $1 - \frac{x^2}{2}$ to approximate $\cos(0.3)$.