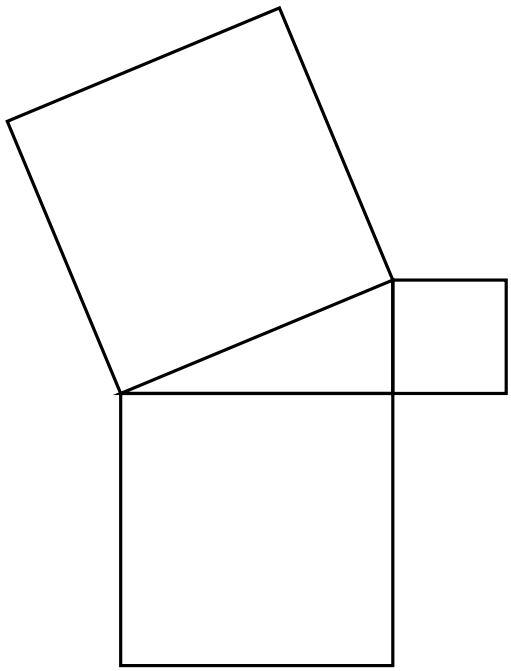
Math 111 - Midterm 1 Review Problems

Here are some review problems for the first midterm. Two thirds of the exam will be based on problems just like these. The rest will be similar to the problems from the workshops and homework.

1. This picture depicts (but does not prove!) a famous theorem from geometry. What theorem is it, and what does it say about the three squares in this picture?



2. Describe in words how to tell if a number is divisible by 300.
3. How many prime numbers are there?
4. What is the difference between a rational and an irrational number.
5. How can you tell that the number $6.\overline{7} = 6.7777...$ is rational?
6. It is a fact that $17 \times 24 = 408$. Use this information to help find 413 mod 24 without a calculator.
7. What are the first 3 positive numbers that are equivalent to -13 modulo 11?
8. Is 67122 divisible by 33? How can you tell without a using a calculator or long division?
9. Explain why calculating mod 10's is so easy.
10. Compute the following moduli (without a calculator)
    (a) 3564 mod 3
    (b) 365723 mod 11

(c) 455417 mod 9

11. If it is 10 AM now, then what time will it be 17 hours from now? What about 37 hours from now?

12. If today is Monday, then what day of the week will it be 100 days from now? What day of the week would it be 777 days from now?

13. If I tell you that 216 days from now will be a Friday, then what day is it today?

14. What are the first 10 prime numbers?

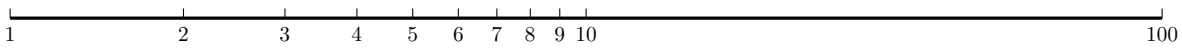
15. Find the following moduli (without a calculator):
    (a) 13 mod 5
    (b) 7 mod 2
    (c) 42 mod 3

16. Why do bar codes have a check digit? What does the check digit do? Explain.

17. Without a calculator, what is log(8)? Explain your reasoning.
    (a) 8.01
    (b) 7
    (c) 0.903
    (d) 1.02

18. What is $\log(10,000)$? What is $\log(0.01)$?

19. Indicate where the numbers 25, 48, and 70/3 belong on the logarithmic scale below.



20. New Jersey contains 2.85% of the United States population. What is New Jersey's standard quota of the 435 seats in the house of representatives.

21. If the U.S. population was 435 million, what would the standard divisor be for the 435 seats in the House of Representatives? What would the standard divisor represent?

22. Explain what the Alabama paradox is. In what year did the original Alabama paradox occur?

23. Which is the only apportionment method where the Alabama paradox is a problem?

24. Suppose that a company needs to apportion a shipment of new 60 computers to its 4 different sites (A, B, C, and D), and the company determines that the standard quotas for the sites are:

Site	A	B	C	D
Quota	5.53	17.61	23.79	13.07

How many computers will each site receive if we use Hamilton's method?

25. A small town has four bus routes. The average number of passengers for each bus route is listed below. The town has 20 buses and would like to apportion the buses so that each route gets a number of buses proportional to the average number of riders.

Bus Routes				
Route	**A**	**B**	**C**	**D**
Average # of riders	432	272	831	465

(a) In this apportionment problem, what are the "seats" and what are the "states"?

(b) What is the total population?

(c) What is the standard divisor?

26. Using modular arithmetic, explain why the divisibility test for 9 works.

27. In any UPC barcode d_1-$d_2d_3d_4d_5d_6$-$d_7d_8d_9d_{10}d_{11}$-d_{12}, the last digit d_{12} is chosen so that

$$[3,1,3,1,3,1,3,1,3,1,3,1] \cdot [d_1,d_2,d_3,d_4,d_5,d_6,d_7,d_8,d_9,d_{10},d_{11},d_{12}] \bmod 10 = 0.$$

Find the last digit (i.e., the check digit) if the rest of the bar code is 1-10510-00222-x.