Change of Basis

I gave an incorrect definition of the change of basis matrix a second time in class Tuesday. Here are two theorems that better explain what is going on.

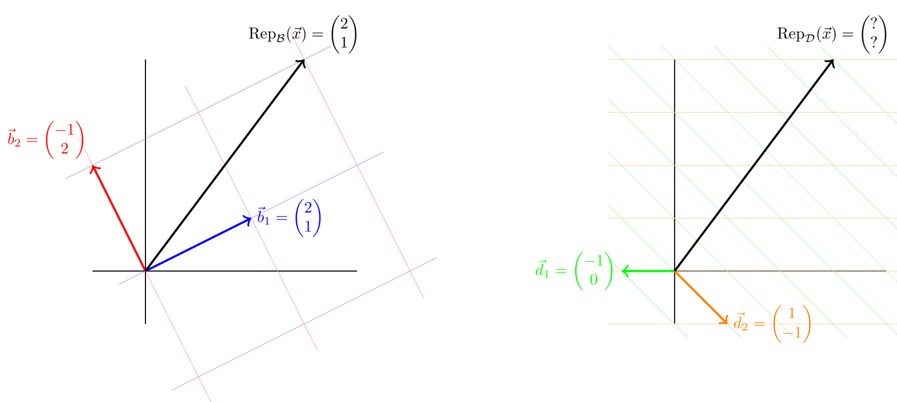
Theorem 1. *Let $B, D \in M^{n\times n}$. If D is invertible, then BD^{-1} is the matrix that transforms the columns of D to the columns of B.*

See if you can prove Theorem 1. It is a very important theorem, but it is not talking about change of basis. Here is the theorem that explains how to change the representation of a vector from one basis to another:

Theorem 2. *Assume that $\mathcal{B}$ and $\mathcal{D}$ are two bases for $\mathbb{R}^n$ and B, D are the matrices with column vectors from $\mathcal{B}$ and $\mathcal{D}$ respectively. For any $\vec{x} \in \mathbb{R}^n$, $Rep_{\mathcal{D}}(\vec{x}) = D^{-1}B * Rep_{\mathcal{B}}(\vec{x})$.*

We call $D^{-1}B$ the **change of basis matrix** from $\mathcal{B}$ to $\mathcal{D}$ because it changes a representation with respect to $\mathcal{B}$ to a representation with respect to $\mathcal{D}$. Notice that the vector $\vec{x}$ does not change, just the representation does.

Example. In the figure below, the same vector $\vec{x}$ is shown with coordinate systems from basis $\mathcal{B} = \langle \vec{b}_1, \vec{b}_2 \rangle$ on the left and basis $\mathcal{D} = \langle \vec{d}_1, \vec{d}_2 \rangle$ on the right.



1. Use the picture to find $\text{Rep}_{\mathcal{D}}(\vec{x})$.
2. Find the change of basis matrix from $\mathcal{B}$ to $\mathcal{D}$.
3. Use the change of basis matrix to find $\text{Rep}_{\mathcal{D}}(\vec{x})$ using Theorem 2. Do you get the same answer you got before?
4. Find the change of basis matrix from $\mathcal{B}$ to $\mathcal{E}_2$.
5. What is $\vec{x}$ represented with respect to the standard basis?
6. Find the matrix that transforms $\vec{b}_1$ to $\vec{d}_1$ and $\vec{b}_2$ to $\vec{d}_2$, and the matrix that undoes that transformation. Are either of these two matrices the change of basis matrix?