Eigenvalues **Math 231 - Workshop**

Recall that the *trace*, $\operatorname{tr}(A)$, of a matrix A is the sum of the entries on the main diagonal.

Theorem 1. *If A is a matrix with n real eigenvalues $\lambda_1, \ldots, \lambda_n$, then* $\det(A) = \lambda_1 \lambda_2 \cdots \lambda_n$ *and* $\operatorname{tr}(A) = \lambda_1 + \lambda_2 + \ldots + \lambda_n$.

The theorem is true for all square matrices, but let's prove it for 2-by-2 matrices.

1. Compute the characteristic polynomial $p(x)$ of $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Use the variable x instead of λ.

2. Since A has two real eigenvalues, the characteristic polynomial can be factored. After you factor it, you get $p(x) = (x - \lambda_1)(x - \lambda_2)$. Of course, you probably don't want to factor your answer from part 1! Instead, expand $p(x) = (x - \lambda_1)(x - \lambda_2)$ (i.e, FOIL.).

3. Match the like terms in two formulas for the characteristic polynomial. What do you notice about the terms with an x? What about the constant terms? Have we proven the theorem above?

4. If I have a 2-by-2 matrix with trace 8 and determinant 15, what are its eigenvalues?

Corollary 1. *If A is a 2-by-2 matrix, then the characteristic polynomial of A is*

$$p(x) = x^2 - \operatorname{tr}(A)x + \det(A).$$

5. Use the corollary above to find the characteristic polynomial and the eigenvalues of $A = \begin{pmatrix} 9 & -12 \\ -2 & -1 \end{pmatrix}$. Then find the eigenvectors.