Definition of a Vector Space

A **vector space** is a set V with the following axioms.

Addition Rules

1. V is **Closed under Addition.** To every pair $\vec{x}, \vec{y} \in V$, there is a vector $\vec{x} + \vec{y}$ in V called the *sum* of $\vec{x}$ and $\vec{y}$.
2. **Addition is Commutative**
$$\vec{x} + \vec{y} = \vec{y} + \vec{x}.$$
3. **Addition is Associative**
$$(\vec{x} + \vec{y}) + \vec{z} = \vec{x} + (\vec{y} + \vec{z}).$$
4. **Additive Identity** there is a unique vector $\vec{0} \in V$ (called the *origin*) such that
$$\vec{x} + \vec{0} = \vec{x} \text{ for all } \vec{x} \in V.$$
5. **Additive Inverses** for every vector $\vec{x} \in V$, there is a unique vector $-\vec{x}$ such that
$$\vec{x} + (-\vec{x}) = \vec{0}.$$

Scalar Multiplication Rules

1. V is **Closed under Scalar Multiplication.** To every vector $\vec{x} \in V$ and every scalar $a \in \mathbb{R}$, there is a unique vector $a\vec{x} \in V$ called the *product* of a and $\vec{x}$.
2. **Scalar Multiplication is Associative**
$$a(b\vec{x}) = (ab)\vec{x}.$$
3. **Scalar Multiplicative Identity**
$$1\vec{x} = \vec{x} \text{ for every vector } \vec{x} \in V.$$
4. **Scalar Multiplication is Distributive** with respect to vector addition:
$$a(\vec{x} + \vec{y}) = a\vec{x} + a\vec{y}.$$
5. **Scalar Multiplication is Distributive** with respect to scalar addition:
$$(a + b)\vec{x} = a\vec{x} + b\vec{x}.$$