Linear Algebra **Midterm 1 Review Problems**

1. Evaluate each of the following:

   (a) $\begin{pmatrix} 1 & 0 \\ 0 & 2 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

   (b) $\begin{pmatrix} 5 \\ -2 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

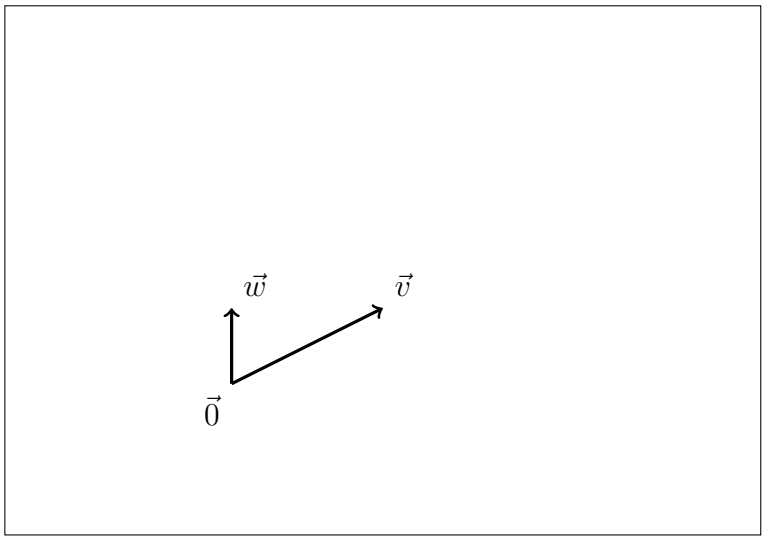
   (c) $||\vec{v}||$ where $\vec{v} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$.

2. Using the space below, draw the following abstract vectors (as arrows beginning at the origin).

   (a) $\vec{v} + \vec{w}$

   (b) $3\vec{v} - \vec{w}$

   (c) $\vec{w} - \vec{v}$



3. Where does the line $\begin{pmatrix} 4 \\ 0 \\ 5 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} t$ intersect the plane $\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} r + \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} s$?

4. Which vector in the set $\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} t \ : \ t \in \mathbb{R} \right\}$ is orthogonal to the vector $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$?

5. Prove that for any vectors $\vec{v_1}, \ldots, \vec{v}_n$ in a vector space V, the set span $\{\vec{v}_1, \ldots, \vec{v}_n\}$ is a subspace.

6. Prove that the set of all solutions of an inhomogeneous system of linear equations can **never** be a vector space.

7. The augmented matrix of a system of linear equations has the following reduced echelon form. Use it to find the general solution of the system of equations (you may assume

that the variables are x_1, x_2, x_3, x_4).

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & -2 & 0 \end{array}\right)$$

8. Use row-reduction to put the following matrix to reduced row echelon form.

$$\begin{pmatrix} 5 & 1 & 1 & 7 \\ 4 & 2 & 1 & 2 \\ 0 & 0 & 3 & 0 \end{pmatrix}.$$

   Show each step.

9. The following three vectors all lie on the same line. Find an equation for the line (with parameter t) and then find the values of t that give each of the three vectors.

$$\vec{u} = \begin{pmatrix} -2 \\ 0 \\ 7 \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}, \quad \vec{w} = \begin{pmatrix} 10 \\ -4 \\ -5 \end{pmatrix}.$$

10. Below is a system of linear equations.

$$4x - 5y = 0$$

$$2y - z = 1$$

$$x + 3z = 0$$

    Re-write the system of equations in the following forms.

    (a) Vector equation form.

    (b) Augmented matrix form.

    (c) Express the system as the claim that a vector is in a given linear span.