Exponential Growth — Math 111

Repeated growth by a constant growth factor is called **exponential growth**. If the growth factor is less than 1, it is called **exponential decay**. When you add a fixed amount at each step, you get **linear growth**.

1. A certificate of deposit (CD) pays 1.2% interest per year. How much will a $3,000 CD be worth after 15 years?

2. From 1980 to 2010, China's average annual real economic growth rate was 9.98%. By what factor did China's economy grow during this 30-year period?

3. Sometimes when we work with percentages we convert the percentage to a growth factor. Other times, we just convert the percentage to a decimal and don't add 1 to make it a growth factor. Explain why we do these two different things and under what circumstances each one is appropriate. *Hint: your answer should mention the difference between a* **"percentage change"** *versus a* **"percentage of something"**.

4. If a sequence of numbers grows by the same factor at each step, we say it **grows exponentially**. If something grows by adding the same amount each step, it **grows linearly**. Which of the following sequences are growing exponentially and which are growing linearly?
   (a) $1, 5, 9, 13, 17, 21, ...$
   (b) $10, 30, 90, 270, ...$
   (c) $1000, 2000, 3000, 4000, ...$

5. Why is adding the same amount each step called linear growth? What does it have to do with lines?

6. Why is multiplying by the same amount each step called exponential growth? What does it have to do with exponents?

7. **Rule of 70** The rule of 70 is a method for estimating how long it will take an investment to double. You take 70 and divide it by the interest rate. For example, if the interest rate is 7% per year, then it will take about $70/7 = 10$ years for the investment to double.

   (a) According to the rule of 70, how long will it take an investment with a 5% annual yield to double?

   (b) If you really earn 5% interest every year, then how much will your investment grow in 14 years as an exact percentage? Does this answer fit with part (a)? Explain.

   (c) Given that China's economy grew by approximately 10% per year, for 30 years, how many times did it double according to the rule of 70?