Factors and Factor-Label Method — Math 111

A **factor** is a number that is multiplied or divided from another number.

Questions

1. When we say that something has doubled, what factor are we talking about?

2. The cost of oil has decreased by a factor of two from this time last year. If it was \$80 per barrel last year, then what is the cost now?

3. In 2000, the population of the United States was 282 million people. In 2015 it was 320 million. By what factor has the population increased?

One special kind of factor is a **conversion factor** that is used to convert numbers measured with one unit into the equivalent number in a different unit. For example, 1 yard is 3 feet so the conversion factor to convert yards into feet is $\left(\frac{3 \text{ feet}}{1 \text{ yard}}\right)$. In words: 3 feet per yard. To convert 9 yards into feet, just multiply by the conversion factor.

$$9 \text{ yards} \times \left(\frac{3 \text{ feet}}{1 \text{ yard}}\right) = 18 \text{ feet}$$

Notice that the units for yards cancel out, leaving only the units for feet.

Questions

1. What is the conversion factor to convert feet back to yards?

2. What is the conversion factor to convert kilometers to meters?

3. What is the conversion factor to convert decimal numbers like 0.6 into percentages?

Conversion factors can be strung together to convert several units at once. This technique is called **factor-label method**. For example, to compute how many seconds there are in one year, just multiply the following conversion factors together:

$$1 \text{ year} \times \left(\frac{365 \text{ days}}{1 \text{ year}}\right) \times \left(\frac{24 \text{ hours}}{1 \text{ day}}\right) \times \left(\frac{60 \text{ minutes}}{1 \text{ hour}}\right) \times \left(\frac{60 \text{ seconds}}{1 \text{ minute}}\right).$$

That is definitely a job for a calculator! Remember that all of the units will cancel leaving an answer that is in seconds. In this case, the answer is 31,536,000 seconds.

Many of the numbers we use every day are actually conversion factors in disguise. For example, if a car drives 50 mph, the speed is a conversion factor for converting time measured in hours into distance in miles.

Questions

1. If a card drives 50 mph for 90 minutes, how far does it drive?

2. If a car drives 50 mph for 5 minutes, how far does it drive?

3. Fuel efficiency is measured in miles per gallon (MPG). For example, a car might get 30 miles per gallon on the highway. Multiplying by 30 MPG is a conversion factor for converting _____ to _____. *Hint: It is not miles to gallons!*

4. How many gallons of gas would it take to drive 100 miles in a car that gets 30 MPG?

Notice that multiplication and division work together the exact same way that addition and subtraction do, with one exception: in addition and subtraction, the numbers have to have the same units, but that is not true with multiplication and division. Multiplication and division combine units to get new units. For example,

Questions

1. A football field is $53\frac{1}{3}$ yards wide and 120 yards long (including the endzones). What is the area of a football field and what are the units that you are using to measure area?

2. How many square feet are in a square yard? *Hint: it is not 3.*

3. How many cubic feet are in a cubic yard?

Extra Practice

These are extra practice problems similar to ones that might appear on a quiz or a test.

1. Suppose that a car drives 360 miles and averages 30 miles per gallon, and gasoline costs \$4 per gallon.
   (a) What is the conversion factor that converts miles to gallons?
   (b) What is the conversion factor that converts gallons to dollars?
   (c) How much does the fuel for this trip cost?

2. A football field is 100 yards long. A yard is 3 feet. A mile is 5280 feet. A marathon is 26.2 miles. Running a marathon would be the same as running the length of how many football fields? Show your conversion factors.

3. Suppose 1 Euro is worth 1.08 Dollars. If a car costs \$20,000 without tax, how many Euros would it cost?

4. If the person from the last problem has 5.65% of her income withheld because of payroll (Medicare and Social Security) taxes, how much will they actually make each year?

5. In English, the following phrases mean different things. Explain what mathematical operation $(+, -, \times, \div)$ is implied by each phrase.
   (a) "Four dozen"
   (b) "Four and a dozen"
   (c) "Four out of a dozen"

6. What does "percent" mean? Hint: what is another phrase that means the same thing as "per" and what does "cent" refer to?

7. Explain the difference between a factor and a term.

8. In algbra, you learned how to factor a polynomial like x^2+3x+2 into the alternate form $(x+2)(x+1)$. Explain why this is called "factoring".

9. Convert each of the following facts into a conversion factor, then put them together to find out how many liters there are in a gallon.

   - Four quarts is a gallon.
   - Four cups is a quart.
   - A cup is 8 fluid ounces.
   - A fluid ounce of water weighs roughly 1 ounce.
   - A pound is 16 ounces.
   - 1 kilogram is approximately 2.2 lbs.
   - 1 kilogram of water is 1 liter.