Proportions   Math 111

In math, the words **ratio**, **fraction**, and **proportion** all mean the same thing, although they have slightly different connotations that mostly have to do with the context where they are used.

Many problems can be solved when you know that two ratios are the same, by setting up a **proportion equation**:

$$\frac{A}{B} = \frac{C}{D}.$$

Remember: an **equation** is a formula with an equals sign. As long as you know any three of the values in a proportionality equation, you can solve for the fourth. Remember: You can do anything you want to an equation, as long as you do it to both sides of the equals sign. Here are things you can do:

- **Multiply both sides by B or D.** Then $B = \dfrac{AD}{C}$ and $D = \dfrac{BC}{A}$.

- **Flip both sides.** $\dfrac{B}{A} = \dfrac{D}{C}$.

- **Cross multiply. (Multiply both sides by both B and D)**

$$\frac{A}{B} = \frac{C}{D} \quad \text{becomes} \quad AD = BC.$$

Questions

1. A large lake contains 400 fish that have been tagged. Researchers go an catch 300 fish from the lake and observe that 19 of the fish have already been tagged. Assuming that fish that are caught are representative of the fish in the whole lake, how many fish are there in the whole lake?

> **Solution:** To set up a proportion equation, start with the analogy:
>
> The # of fish in the sample with tags is to the # of fish in the lake with tags as the total # of fish in the sample is the total # of fish in the lake. This becomes the proportion equation:
>
> $$\frac{19}{400} = \frac{300}{x}$$
>
> where x is the unknown number of fish in the lake. You can solve for x by cross-multiplying to get
>
> $$19x = 120{,}000$$
>
> and then $x = 120{,}000/19 = 6315.79$. So there are approximately 6,316 or so fish in the lake.

Percents

All percents can be thought of in terms of proportion equations. Remember, the symbol %
is not decoration, it literally means 'divide by 100'. When working with percents, remember,

$$\frac{\text{part}}{\text{whole}} = \frac{x}{100}$$

where x is the percent without the % symbol.

1. The sale price of a mattress is $240. If this price is 60% of the regular price, then what
   is the regular price?

Solution: The part is $240. We don't know the whole, so use a variable like x. The
proportion equation is
$$\frac{240}{x} = \frac{60}{100}.$$

Cross-multiplying gives
$$24,000 = 60x.$$

Then dividing both sides by 60 gives

$$x = 400.$$

So the regular price is $400.

Additional Practice

1. 27 is what percent of 150?

2. Last year, scientists tagged 80 deer in a large national park. This year, when they
   capture a sample of 70 deer, 9 already have tags. If the sample is representative of the
   deer in the park, then estimate how many deer there are.

3. A 10 ft long metal rod weighs 192 lbs. How much would a 3 ft section of the rod weigh?

4. Can you solve the previous problem a different way, using conversion factors in stead of
   a proportion equation?