In math, the words **ratio**, **fraction**, and **proportion** all mean the same thing. When two ratios are the same, you can solve for an unknown part by setting up a **proportion equation**:

$$\frac{a}{b} = \frac{c}{d}.$$

Remember: an **equation** is a formula with an equals sign. As long as you know any three of the values in a proportion equation, you can solve for the fourth. The key is this simple rule from algebra: You can do anything you want to an equation, as long as you do it to both sides of the equals sign. Here are some things you can do in a proportion equation:

Multiply both sides by b	$\frac{a}{b} = \frac{c}{d}$	becomes	$a = \frac{bc}{d}$
Flip both sides	$\frac{a}{b} = \frac{c}{d}$	becomes	$\frac{b}{a} = \frac{d}{c}$
Cross multiply (multiply both sides by bd)	$\frac{a}{b} = \frac{c}{d}$	becomes	ad = bc

Example: Eratosthenes Measures the Earth

Proportions

Math 111

In 240 B.C.E., the Greek mathematician Eratosthenes was able to measure the circumference of the Earth. He knew that at noon on the summer solstice, the light from the sun shone straight down to the bottom of a deep well in Syene (now Aswan) in Egypt. In his native Alexandria, Eratosthenes could measure the shadow cast by the sun at noon on the solstice.

1. Eratos thenes measured the angle θ to be 7.2°. Complete the analogy below:

 $\underline{7.2^{\circ}}$ is to $\underline{360^{\circ}}$ as ______ is to the circumference of the Earth.

2. Change the analogy above to a **proportion equation** and solve for the circumference of the Earth.



Source: NOAA Ocean Service Education

3. A "stades" was an ancient unit for measuring distance. One stades is about 1/10 of a mile. Was Eratosthenes close to the correct value of the circumference of the Earth? (The true value is 24,900 miles.)

Percents

The words **percentage** and **percent** (when used as a noun) mean the same thing, so you can use whichever sounds better in context. Be careful with the word **percentile**, it has a special meaning that we'll talk about later. Any percentage is really a proportion:

$$x\% = \frac{x}{100} = \frac{\text{part}}{\text{whole}}.$$

The symbol % is not decoration, it literally means 'divide by 100'. So 50% is not 50, it is $\frac{50}{100}$.

4. What was Eratosthenes estimate as a percentage of the true circumference of Earth? Hint: start with a proportion equation based on the analogy:

Eratosthenes' circumference is to the true circumference as ______ is to 100.

5. Was Eratosthenes estimate too big or too small? How far off was he, as a percent?



Figure: Eratosthenes' map of the known world