Homework 2 - Math 254 **Name:** _____

Due in class Wednesday, February 2.

1. Express each statement in one of the forms $P \wedge Q$, $P \vee Q$, or $\sim P$. Be sure to also state exactly what statements P and Q stand for.

   (a) $x \in A - B$

   (b) It will not rain today, but it will rain tomorrow.

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2. Convert each of the following sentences into a sentence having the form "*if P, then Q.*"

   (a) In order for a function to be differentiable, it must be integrable.

   (b) Every nonempty set has more than one subset.

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3. Convert each of the following sentences into a sentence having the form "*if P, then Q.*"

   (a) A continuous function has a critical point if it has more than one root.

   (b) An integer is divisible by 10 only if it is divisible 5.

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4. Use a truth table to show that $\sim(P \Rightarrow Q)$ is logically equivalent to $P \wedge \sim Q$.

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5. Sketch the following set in the (x, y)-plane: $\{(x, y) \in [1, 2] \times [1, 3] : x = 1 \text{ or } y \in \{1, 2, 3\}\}$.

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6. Write the following as English sentences. Say whether they are true or false.

   (a) $\forall x \in \mathbb{R}, \exists n \in \mathbb{N}, x < n$.

   (b) $\exists n \in \mathbb{N}, \forall X \in \mathcal{P}(N), |X| < n$.

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7. Translate each of the following sentences into symbolic logic.

   (a) For any positive real number x, there is a positive rational number less than x.

   (b) Every odd degree polynomial p has a real number x such that $p(x) = 0$.

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8. Negate the following logical expressions.

   (a) $\forall x > 0, \exists y > 0, xy = 1$.

   (b) $\forall X \in \mathcal{P}(\mathbb{N}), X \in \mathbb{R}$.

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9. Negate the following sentences.

   (a) If x is rational then $\sqrt{x}$ is rational.

   (b) For every $\epsilon > 0$, there is an $M \in \mathbb{R}$ such that $|f(x)| < \epsilon$ whenever $x > M$.

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10. Determine whether the statements $(P \Rightarrow (Q \wedge \sim R))$ and $\sim(P \wedge (Q \Rightarrow R))$ are logically equivalent. Explain how you can tell.
