Homework 3 - Math 254 **Name:** _____

Due in class Wednesday, February 9.

1. How many different lists of length 4 can be made from the symbols A, B, C, D, and E if...

   (a) ...repetitions are allowed?

   (b) ...repetitions are not allowed?

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2. Consider lists made from the symbols A, B, C, D, E, with repetition allowed.

   (a) How many such length-5 lists have at least one letter repeated?
   Hint: First figure out how many length-5 list have no repeats.

   (b) How many such length-6 lists have at least one letter repeated?

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3. A set A has the property that $|\{X \in \mathcal{P}(A) : |X| = 3\}| = 20$. Find $|A|$.

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4. A binary string is a list of the symbols 0 and 1.

   (a) How many 8-digit binary strings have exactly four 1s?

   (b) How many 7-digit binary strings have an odd number of 1s?

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5. What is the fewest number of times you must roll a six-sided dice before you can be assured that 5 or more of the rolls resulted in the same number?

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6. In the game Twenty Questions, one person picks a secret word and the other player (the guesser) can ask 20 Yes/No questions to try to figure out the secret word. Assuming that the secret word is in a standard dictionary which has about 500,000 words, is it always possible for the guesser to figure out the secret word before running out of questions? Explain why.

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7. Find three sets A_1, A_2, A_3 such that $A_1 \cap A_2 \cap A_3 = \varnothing$, but $|A_1 \cup A_2 \cup A_3| \neq |A_1| + |A_2| + |A_3|$.

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8. Give a combinatorial explanation of the following fact (which is true for all $n \in \mathbb{N}$).

$$\binom{2n}{2} = 2\binom{n}{2} + n^2.$$

Hint: If you have a class with n boys and n girls, think about the different ways you could choose two kids from the whole class.
