Homework 5 **Math 254**

Due in class Friday, February 25. Make sure to copy the proposition for each proof and then clearly indicate where your proof starts and ends.

Prove the following statements with a contrapositive proof.

1. Let $x, y \in \mathbb{Z}$. If both xy and $x + y$ are even, then x and y are even.
2. Suppose $a \in \mathbb{Z}$. If a^2 is not a multiple of 4, then a is odd.

Prove the following statements using either a direct or a contrapositive proof.

3. Suppose $x, y, z \in \mathbb{Z}$ and $x \neq 0$. If $x \nmid yz$, then $x \nmid y$ and $x \nmid z$.
4. Let $x \in \mathbb{Z}$ and $n \in \mathbb{N}$. If x divided by n has a remainder r, then $x \equiv r \pmod{n}$. Hint: Use the division algorithm (see Fact 1.5 on page 30 in the book).
5. If $a, b \in \mathbb{Z}$, then $(a + b)^3 \equiv a^3 + b^3 \pmod{3}$.