Homework 6 **Math 254**

Due in class Friday, March 4. Make sure to copy the proposition for each proof and then clearly indicate where your proof starts and ends.

Prove the following statements using a proof by contradiction.

1. Let $a, b, c \in \mathbb{Z}$. If $a^2 + b^2 = c^2$, then a or b is even.

2. Suppose that $a, b \in Z$. If $4 \mid (a^2 + b^2)$, then a and b are not both odd.

Prove the following statements using any method.

3. Suppose $a \in \mathbb{Z}$. If $a \equiv 4 \pmod 5$, then $a^2 \equiv 1 \pmod 5$

4. Let $a, b, n \in \mathbb{Z}$ with $n > 0$. If $a \equiv b \pmod n$, then $\gcd(a, n) = \gcd(b, n)$.
   Hint: *If $d = \gcd(a, n)$, then you need to prove two things. First, prove that d also divides b, and second prove that d is the largest divisor of b and n.*