**Homework 10** **Math 254**

*Due in class Friday, April 8. Make sure to copy the proposition for each proof and then clearly indicate where your proof starts and ends.*

1. For the Fibonacci sequence, prove that $F_{n+2} = 2F_n + F_{n-1}$ for every $n \in \mathbb{N}$.

2. Prove that $7 \mid (8^n + 6)$ for every integer $n \geq 0$.

3. Suppose that $a_1 = \sqrt{2}$, $a_2 = \sqrt{2+\sqrt{2}}$, $a_3 = \sqrt{2+\sqrt{2+\sqrt{2}}}$, ..., and in general $a_{n+1} = \sqrt{2+a_n}$. Prove that $a_n < 2$ for all $n \in \mathbb{N}$.

4. In calculus, the power rule says that $\dfrac{d}{dx}x^n = nx^{n-1}$ for every $n \in \mathbb{N}$. Use mathematical induction to prove the power rule. Hint: You will need to use some other facts from calculus like the product rule: $(fg)' = f'g + fg'$. It is okay to use these other facts without proof.