Homework 12 **Math 254**

Due in class Monday, April 25.

1. Let $f : A \to B$ be a function. Let $Y, Z \subseteq B$. Prove that $f^{-1}(Y \cap Z) = f^{-1}(Y) \cap f^{-1}(Z)$.
2. Is the function $f([x]) = [x^3]$ a bijection on $\mathbb{Z}_7$? Prove your answer.
3. Let $f : A \to B$ and $g : B \to A$ be functions such that $f \circ g = i_B$ where i_B is the identity function on B. Prove that f is onto.
4. Prove that if $f : A \to B$ is one-to-one, then $X = f^{-1}(f(X))$ for every $X \subseteq A$.
5. **Extra credit.** Prove the converse of the claim in problem 4.