Final Exam **Math 254**

Due at noon on Friday, May 6. You can use your notes and the textbook, but you should not consult other sources (including other students). If you have questions, ask me. Make sure to copy the proposition for each proof and then clearly indicate where your proof starts and ends. You grade will be based on your best five solutions out of the six problems below.

Prove the following statements.

1. Suppose $a, b, c \in \mathbb{Z}$ and $a^2 \mid b$ and $b^2 \mid c$. Prove that $a^4 \mid c$.
2. The symbol $\mid$ is a relation on $\mathbb{Z}$. Recall that $a \mid b$ when b is divisible by a. Prove that $\mid$ is transitive.
3. Prove that the divisibility relation $\mid$ is not an equivalence relation.
4. Prove that if $n \in \mathbb{N}$, then $\dfrac{1}{2!} + \dfrac{2}{3!} + \ldots + \dfrac{n}{(n+1)!} = 1 - \dfrac{1}{(n+1)!}$.
5. Let P be a partition of a set A. Use P to define a relation on A as follows: $x \sim_P y$ if and only if x and y are both elements of the same part $X \in P$. Prove that $\sim_P$ is an equivalence relation on A.
6. Let $f : A \to B$ be an onto function. Prove that $f(f^{-1}(Y)) = Y$.