**Interpolation** **Math 342**

Consider the points $(0,0), (1,1), (2,-2), (3,3)$.

1. Find a formula for a 3rd degree interpolating polynomial $p_3$ using the Lagrange basis for the points above.

2. Use divided differences to find the formula for $p_3$ in the Newton basis.

3. What is the Vandermonde system for these four points? You don't need to solve the system.

4. (**Extra Credit**) The function $f(x) = -x\cos(\pi x)$ passes through the four points above. Unfortunately the interpolating polynomial from problems 1 & 2 is not a good approximation for $f$. The Hermite interpolating polynomial is more accurate. One way to find the Hermite polynomial is to use the Lagrange basis functions $l_i(x)$ in this formula:

$$H_{2n+1}(x) = \sum_{i=0}^{n} y_i h_i(x) + \sum_{i=0}^{n} y_i' \tilde{h}_i(x)$$

where

$$h_i(x) = (1 - 2(x - x_i)l_i'(x_i))(l_i(x))^2$$
$$\tilde{h}_i(x) = (x - x_i)(l_i(x))^2,$$

and $y_i'$ is the derivative of $f$ at each node $x_i$. Use this approach to find the Hermite interpolating polynomial $H_7$ with the nodes $x_0 = 0, x_1 = 1, x_2 = 2, x_3 = 3$.