LU Decomposition **Math 342**

The LU decomposition is a way to row-reduce a matrix $A \in \mathbb{R}^{m\times n}$ and keep track of the steps. The result is two matrices $L \in \mathbb{R}^{m\times m}$ and $U \in \mathbb{R}^{n\times n}$ such that $A = LU$. With these two matrices, it is much easier to computationally solve any linear equation $Ax = b$. The matrix U in the decomposition is the echelon form of A (with no row swaps or scaling), and

$$L = \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ L_{ij} & & 1 \end{pmatrix} \text{ where } L_{ij} = \begin{pmatrix} \text{the multiple of row } j \text{ that was } \underline{\text{subtracted}} \\ \text{from row } i \text{ during row-reduction} \end{pmatrix}.$$

Notice that if you add a positive multiple of row i to row j during row-reduction, then the corresponding entry of L will be negative.

1. Find the LU decomposition for $A = \begin{pmatrix} 1 & 4 \\ 4 & 1 \end{pmatrix}$.

2. Use the LU decomposition above to solve $Ax = \begin{pmatrix} 0 \\ 30 \end{pmatrix}$.

3. Find the LU decomposition for the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$.