## Runge-Kutta Method

The fourth order Runge-Kutta method (RK4) is commonly used to numerically approximate the solution of an initial value problem

$$\frac{dy}{dt} = f(t, y)$$
 with  $t \in [a, b]$  and initial condition  $y(a) = y_0$ .

The formula for updating the y-values in RK4 is

$$y_{i+1} = y_i + \frac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4)$$

where

$$k_1 = f(t_i, y_i),$$

$$k_2 = f(t_i + h/2, y_i + hk_1/2),$$

$$k_3 = f(t_i + h/2, y_i + hk_2/2),$$

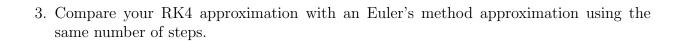
$$k_4 = f(t_i + h, y_i + hk_3).$$

1. Write a function in Python to implement RK4.

2. Use your RK4 function to approximate the solution to the IVP

$$\frac{dy}{dt} = \frac{y}{t} - \left(\frac{y}{t}\right)^2$$

on the interval [1,2] with initial condition y(1) = 1.



4. You should include graphs of both your RK4 and Euler's method approximations.

5. The exact solution to this IVP is  $y(t) = \frac{t}{\ln t + 1}$ . Find the absolute error in the RK4 approximation of y(2) and compare it with the absolute error in the Euler's method approximation (both with n = 10).