Runge-Kutta Method **Math 342**

The fourth order Runge-Kutta method (RK4) is commonly used to numerically approximate the solution of an initial value problem

$$\frac{dy}{dt} = f(t, y) \quad \text{with } t \in [a, b] \text{ and initial condition } y(a) = y_0.$$

The formula for updating the y-values in RK4 is

$$y_{i+1} = y_i + \tfrac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4)$$

where

$$\begin{aligned} k_1 &= f(t_i, y_i), \\ k_2 &= f(t_i + h/2, y_i + hk_1/2), \\ k_3 &= f(t_i + h/2, y_i + hk_2/2), \\ k_4 &= f(t_i + h, y_i + hk_3). \end{aligned}$$

1. Write a function in Python to implement RK4.

2. Use your RK4 function to approximate the solution to the IVP

$$\frac{dy}{dt} = \frac{y}{t} - \left(\frac{y}{t}\right)^2$$

on the interval $[1, 2]$ with initial condition $y(1) = 1$.

3. Compare your RK4 approximation with an Euler's method approximation using the same number of steps.

4. You should include graphs of both your RK4 and Euler's method approximations.

5. The exact solution to this IVP is $y(t) = \dfrac{t}{\ln t + 1}$. Find the absolute error in the RK4 approximation of $y(2)$ and compare it with the absolute error in the Euler's method approximation (both with $n = 10$).