Taylor Polynomials **Math 342**

1. Suppose you use the Taylor polynomial of degree $n = 20$ (centered at zero) to approximate the function $f(x) = e^x$. What does Taylor's theorem say the worst case error would be if you used it to compute e^6? (You can estimate $e \approx 2.72$ in the error formula).

2. Write a Python program to find and sum the terms of the 20th degree Taylor polynomial to approximate e^6. Instead of writing a for-loop, I recommend using a **list-comprehension**:

   [*expression* **for** *item* **in** *iterable*].

   Here is an example:

```
from scipy.special import factorial
sum([1/factorial(n) for n in range(10)])
```

3. Use the `math.exp` function in the Python math library to find the "exact" value of e^6. Compare this with your Taylor polynomial approximation.
   (a) What is the absolute error in your approximation?
   (b) What is the relative error in your approximation?

4. Adjust your program to find the 20th degree Taylor polynomial approximation to find e^{-6}.

5. Compare your answer to the actual value of e^{-6}.
   (a) What is the absolute error in your approximation?
   (b) What is the relative error in your approximation?

6. Compare the following:
   (a) The Maclaurin polynomial approximation for $\sin(4\pi)$ (you can pick the degree, as long as it is at least 20).

   (b) $\sin(4\pi)$ according to Python (using the `sin()` function and `pi` from the `math` library).

   (c) The actual value of $\sin(4\pi)$.

7. Use the Maclaurin series for $\cos x$ to find the Maclaurin series for $\cos\sqrt{x}$. Then integrate to find the Maclaurin series for $\int \cos\sqrt{x}\,dx$.

8. Use Python to approximate $\int_0^1 \cos\sqrt{x}\,dx$.